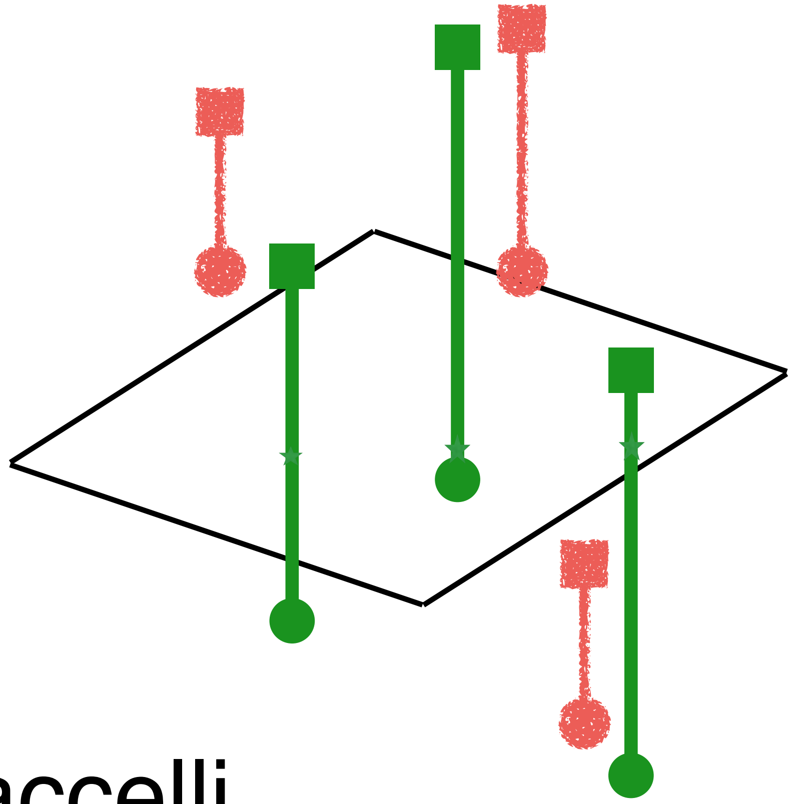
Spatial Birth-Death Model for Wireless Networks



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UT Austin

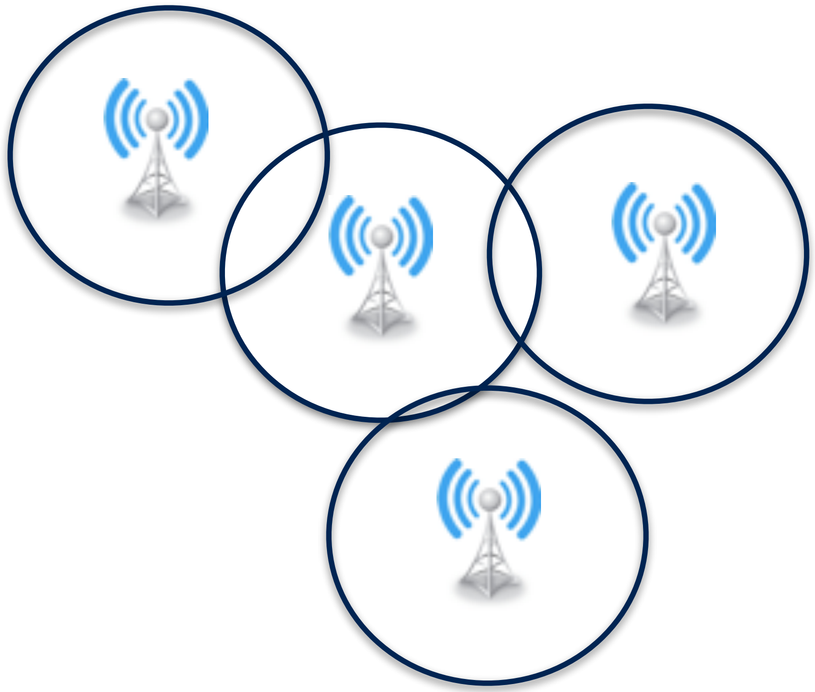
Outline

- Motivation and Background.
- Math Model - Interacting Particle System.
- Summary of Results.
- Further Questions.

Background and Motivation

- Model for a wireless network that captures ***precisely***
  - Interactions in Space

    (Interference)



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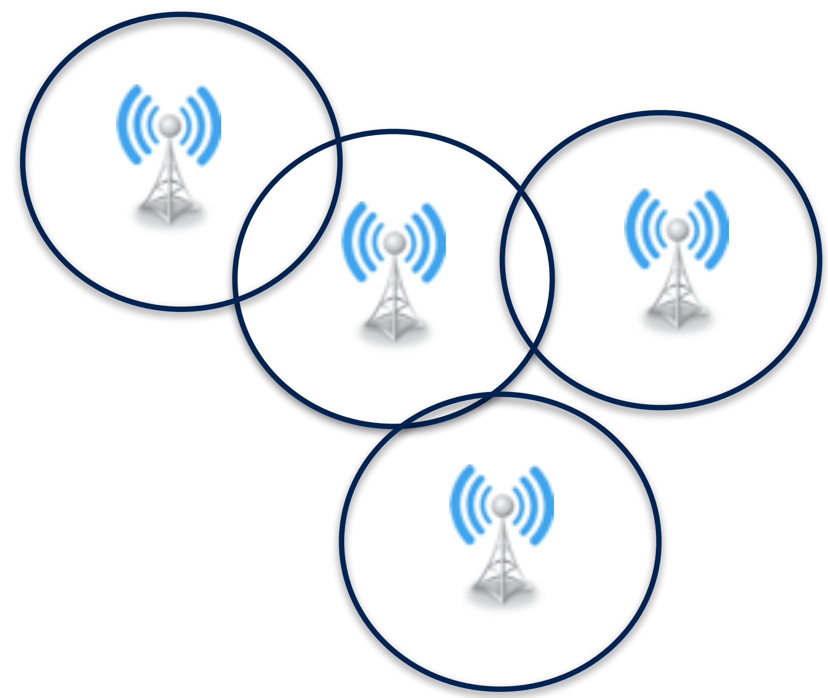

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    - Interactions in Time due to randomness in traffic.

Background - Ad-Hoc Wireless Networks

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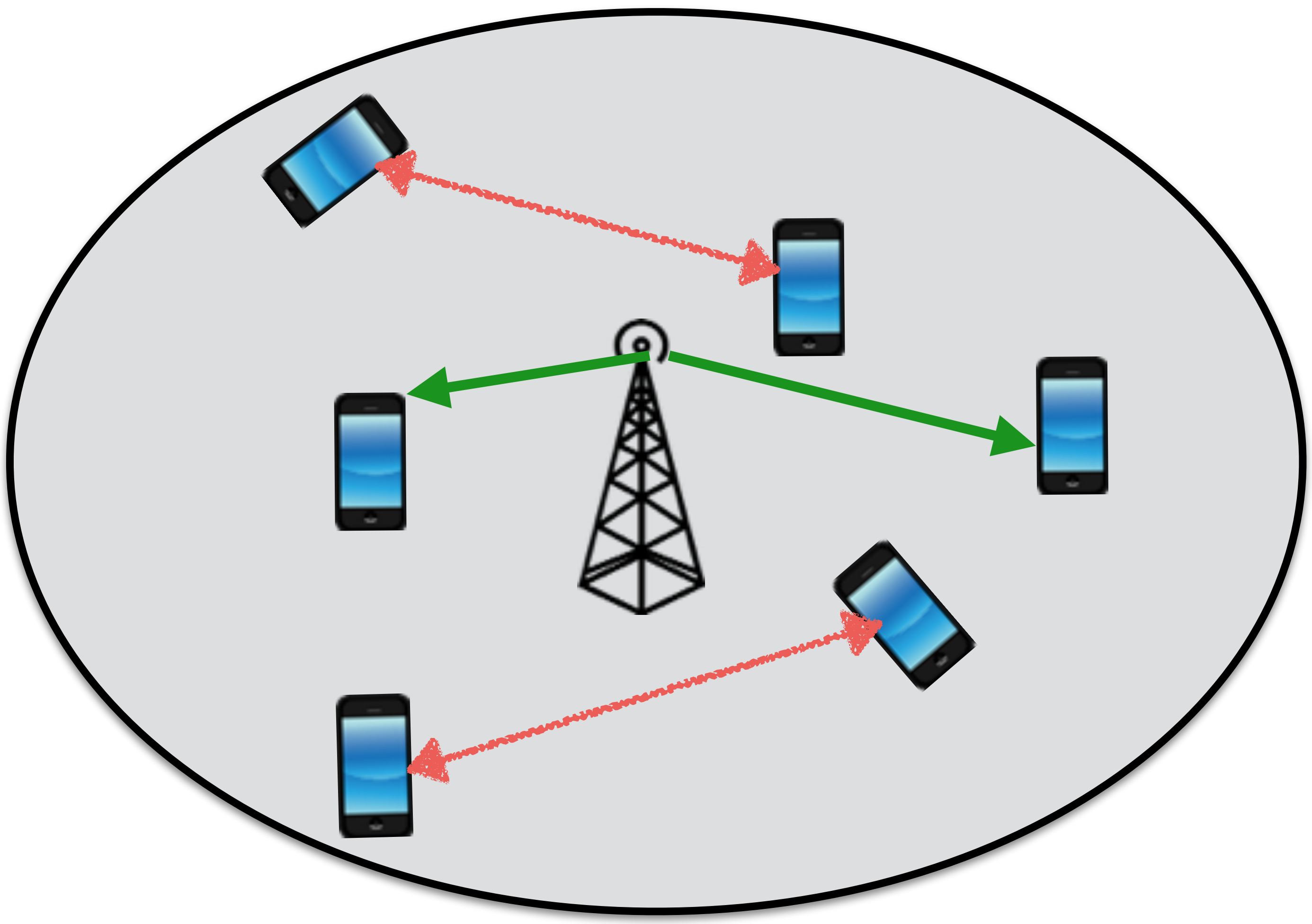
- Main engineering application - Overlaid D2D networks.



- Increasing popularity of D2D as means to offload some cellular traffic. [Dhillon, 15], [Lee, Lin, Andrews, Heath 15], [Lu, DeVeciana 15]

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   (Does not precisely capture interactions through traffic arrivals)

2. Flow-based queuing models (for ex. [Bonald,Proutiere 06])
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A caricature framework that captures the interactions over space and time.

Stochastic Network Model - Preliminaries

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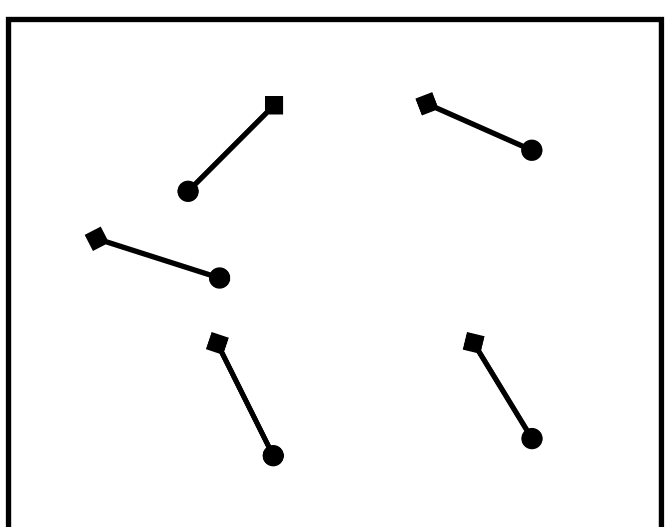
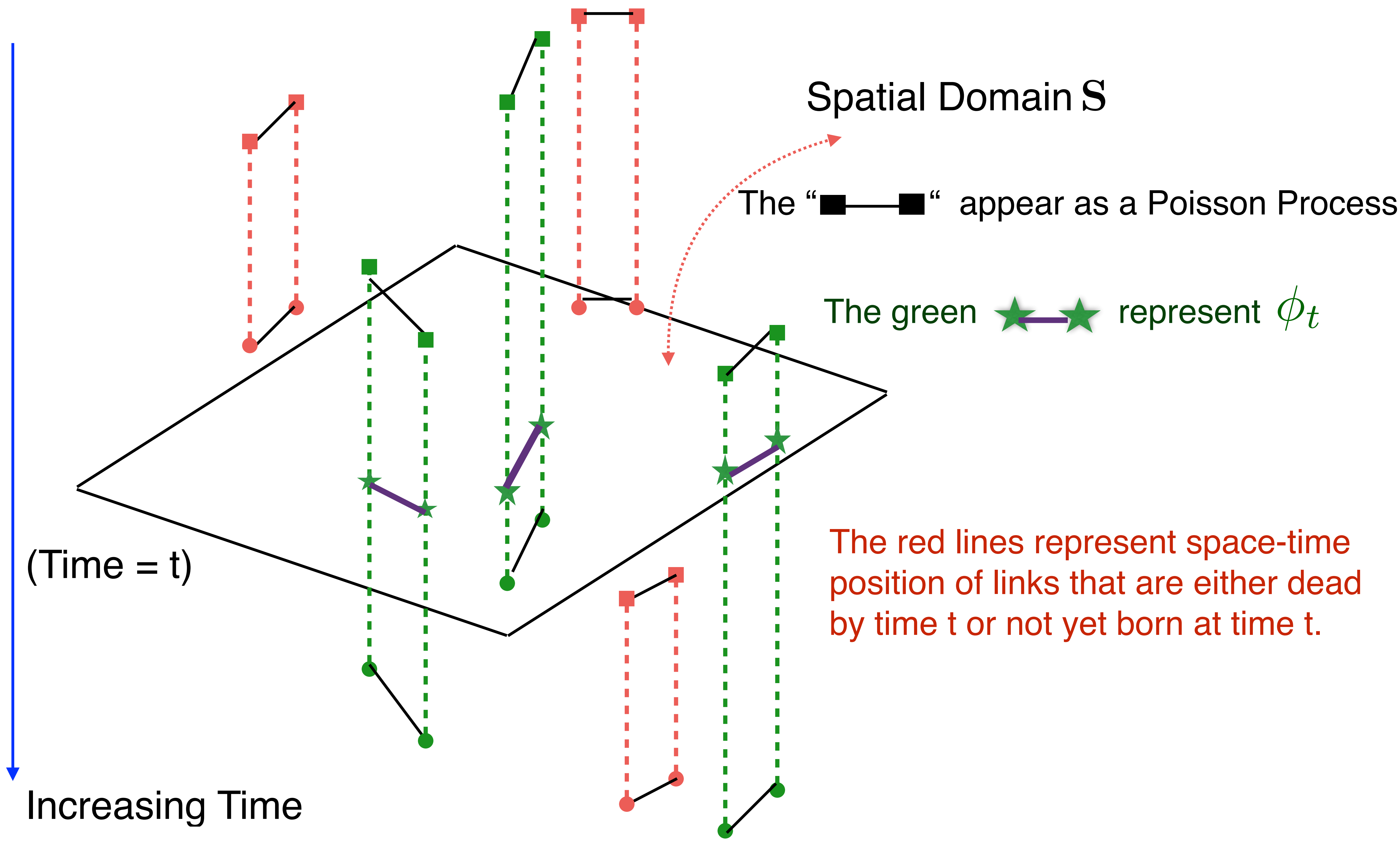
- ϕ_t : The configuration of links present in the system at time t



$$\phi_t = \{(x_1, y_1), (x_2, y_2), \cdots, (x_{N_t}, y_{N_t})\}$$

(Configuration at time t).

Line Segment - Arrival Departure Schematic



Basic Notation

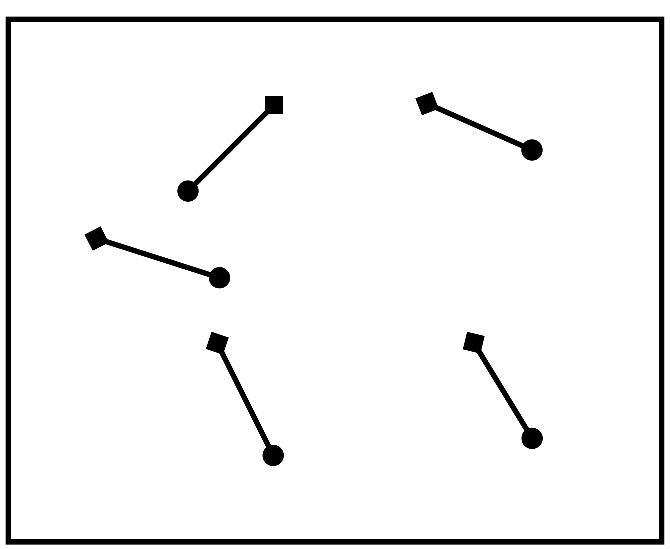
- ϕ_t : The configuration of links present in the system at time t

$\phi_t^{Rx} = \{x_1, x_2, \cdots, x_{N_t}\}$ The set of receivers at time t

$\phi_t^{Tx} = \{y_1, y_2, \cdots, y_{N_t}\}$ The set of transmitters at time t

$||x_i - y_i|| = T \quad \forall i$ $\qquad Tx(x_i) = y_i$

Note - N_t is a random variable depending on the dynamics



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Stochastic Network Model - Dynamics

- Links arrive in the network as a PPP on $\mathbb{R} \times \mathbf{S}$ with intensity λ
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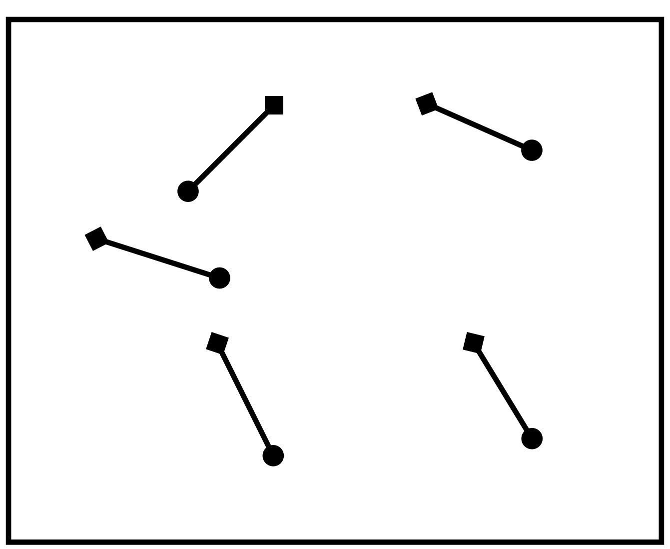
Speed of file transfer ?



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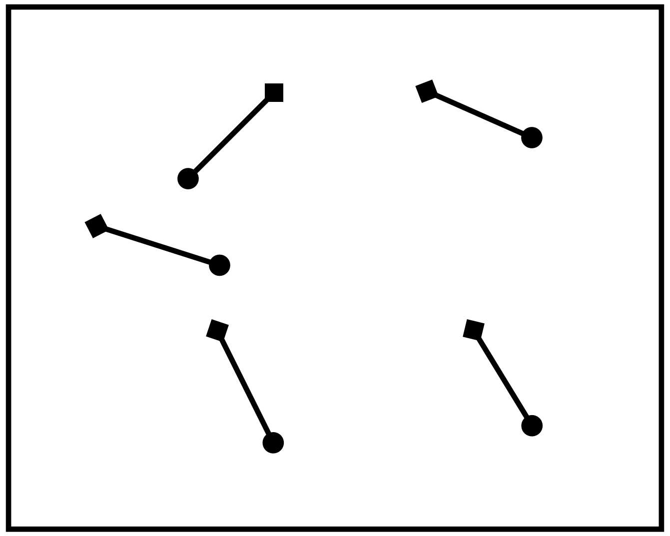
Speed of file transfer ?

Given by the instantaneous Shannon Rate seen at each point.



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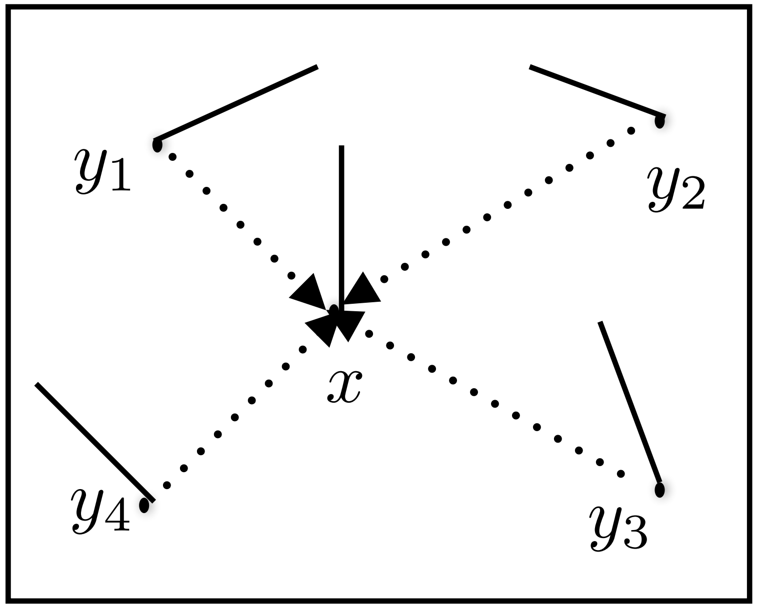
Stochastic Network Model - Dynamics

- Interference seen at point x due to configuration ϕ

$$I(x,\phi) = \sum_{y \in \phi^{Tx} \setminus Tx(x)} l(||y-x||)$$ (distance measured on the torus).

$l(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ called the 'path-loss function'.



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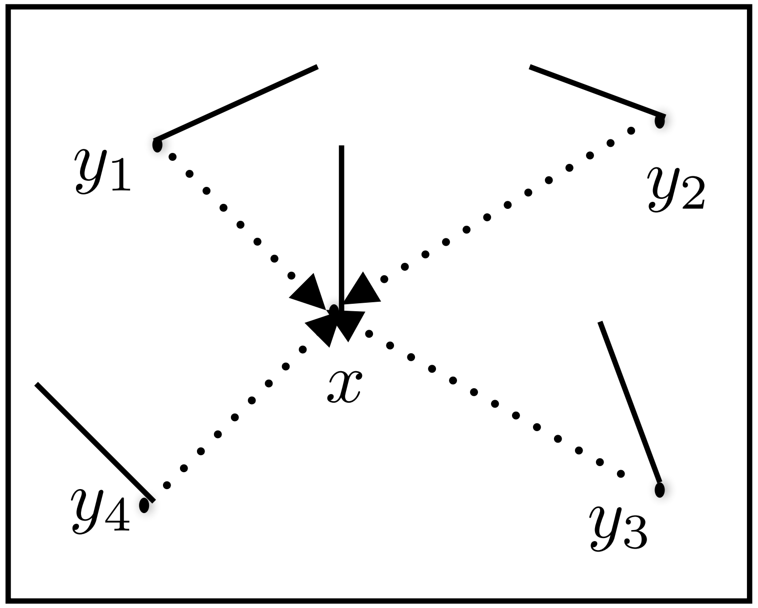
- Speed of file-transfer to point x in configuration ϕ

$$R(x,\phi) = C \log_2 \left(1 + \frac{l(T)}{N_0 + I(x,\phi)}\right)$$ bits per second

A deterministic function of the configuration



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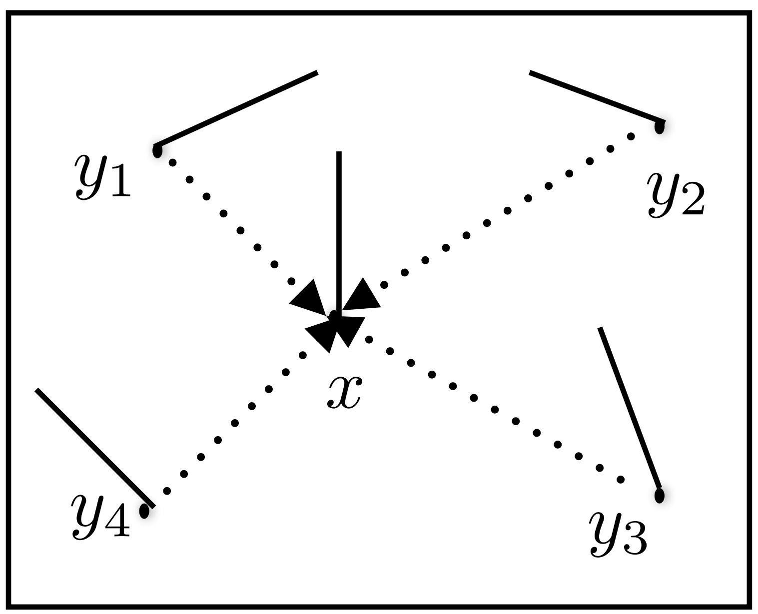
Stochastic Network Model - Dynamics

- The speed of file transfer by link at location x in configuration ϕ

$$R(x,\phi) = C\log_2\left(1 + \frac{l(T)}{N_0 + I(x,\phi)}\right) \text{ bits per second}$$



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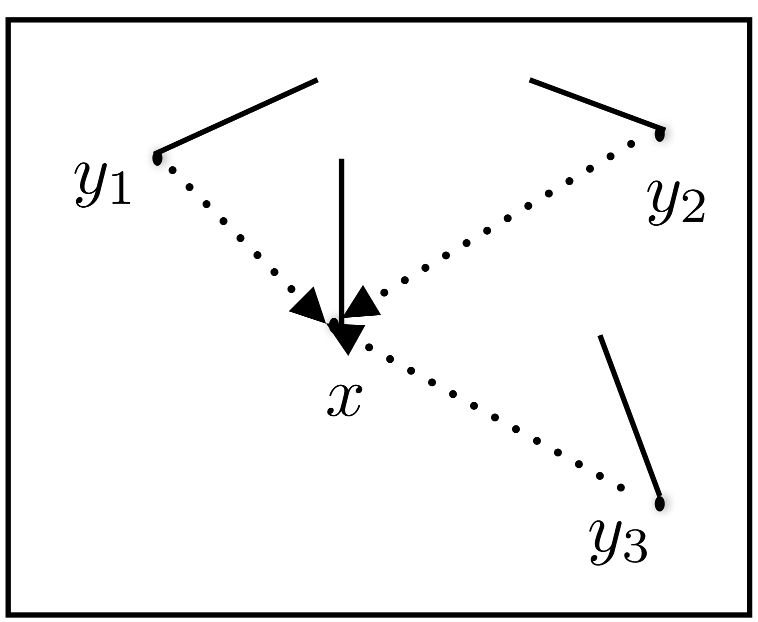
$$R(x,\phi_1) \geq R(x,\phi_2)$$



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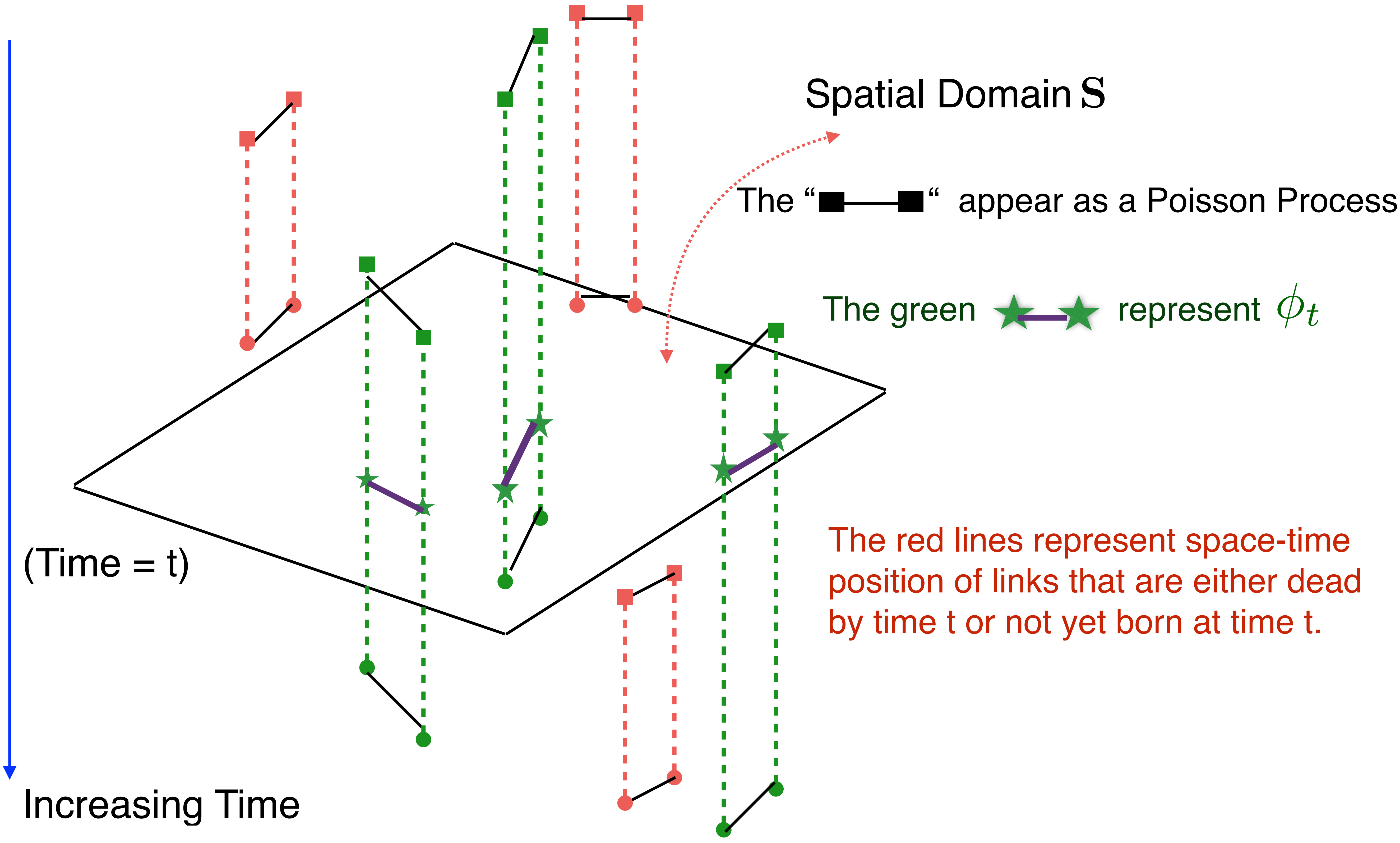
The Problem Statement

- A link 'born' at location x_p and time b_p with file-size L_p leaves the system ('dies') at time $d_p = \inf \left\{ u \geq b_p : \int_{t=b_p}^{u} R(x_p, \phi_t) dt \geq L_p \right\}$ where $R(x, \phi) = C \log_2 \left(1 + \frac{l(T)}{N_0 + I(x, \phi)} \right)$ and ϕ_t is the set of links "alive" at time t.

Spatial Birth-Death Process since -

- Arrivals from the Poisson Rain
- Departures happen after file transfer

Spatial Birth-Death (SBD) Model

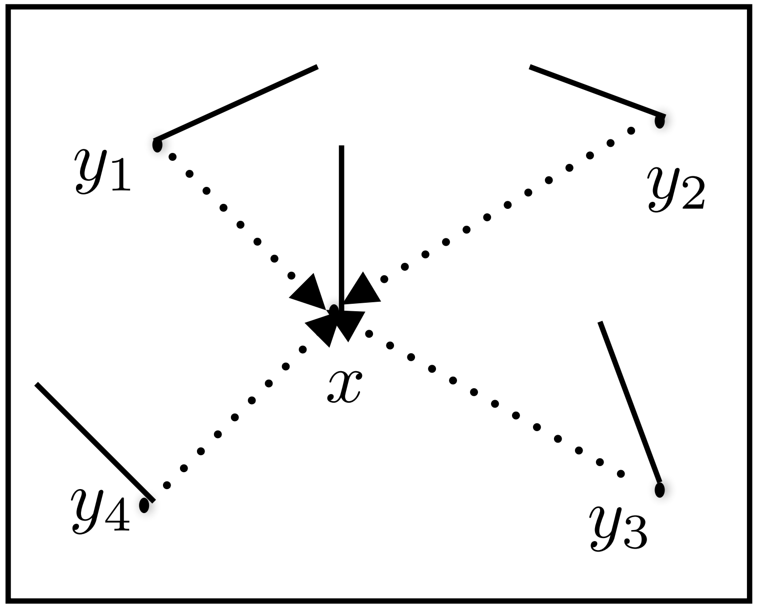


SBD Model - Interacting Particle System

A caricature framework that accounts for spatio-temporal interactions.

The rest of the talk - present results on this model

Conclude with questions on how to enrich the framework to cover different aspects and applications.



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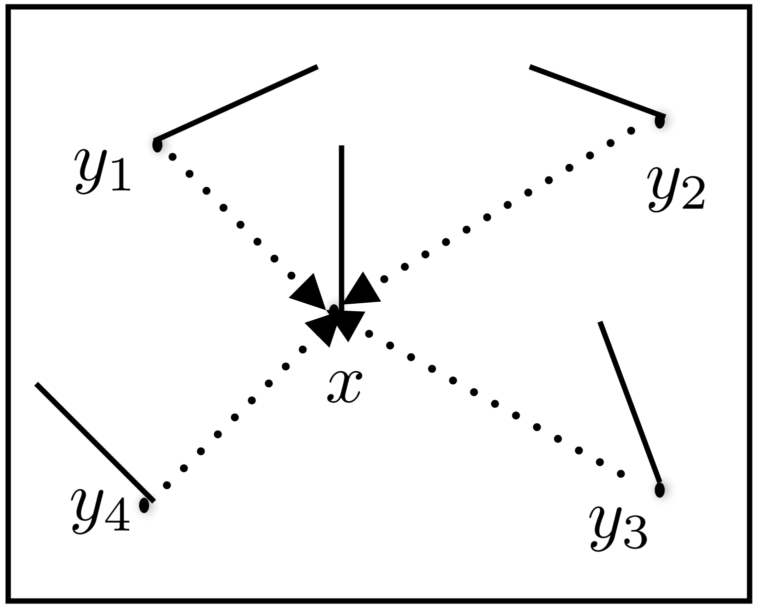
Stochastic Network Model - Details

Model Assumptions.

- $N_0 > 0$ Needed to avoid the corner case of when interference is 0.
- $\mathbf{S} = [-Q,Q] \times [-Q,Q]$ is a compact set.
- $l(r) < \infty$, $\forall r > 0$ Want rate to be non-zero.

The statistical assumptions imply that ϕ_t is a Markov Process on the set of marked simple counting measures on the set $\mathbf{S}$



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However, one can think of a model with 'fast fading' and study it.

$$R(x,\phi) = C\mathbb{E}_h\left[\log_2\left(1+\frac{h_0 l(T)}{N_0+\sum_{y\in\phi^{Tx}\setminus\{T(x)\}} h_y l((||y-x||)}\right)\right]$$

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- We study the simplest scheduling (bandwidth-allocation) i.e. ALOHA.

- We do not consider the interaction of links through intelligent MAC layer scheduling in addition to physical layer interference.

For example, each point measures the interference, and decides to be active only when the interference is below a threshold.

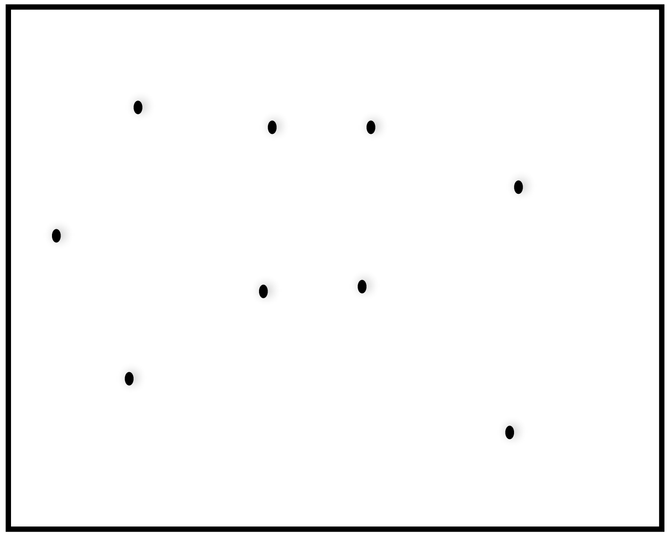
SBD Model - Special Case

Case when T=0.
(The case when link lengths are very small compared to network size.)

The wireless dynamics is evolution of points.

The qualitative features (mathematically) are retained.

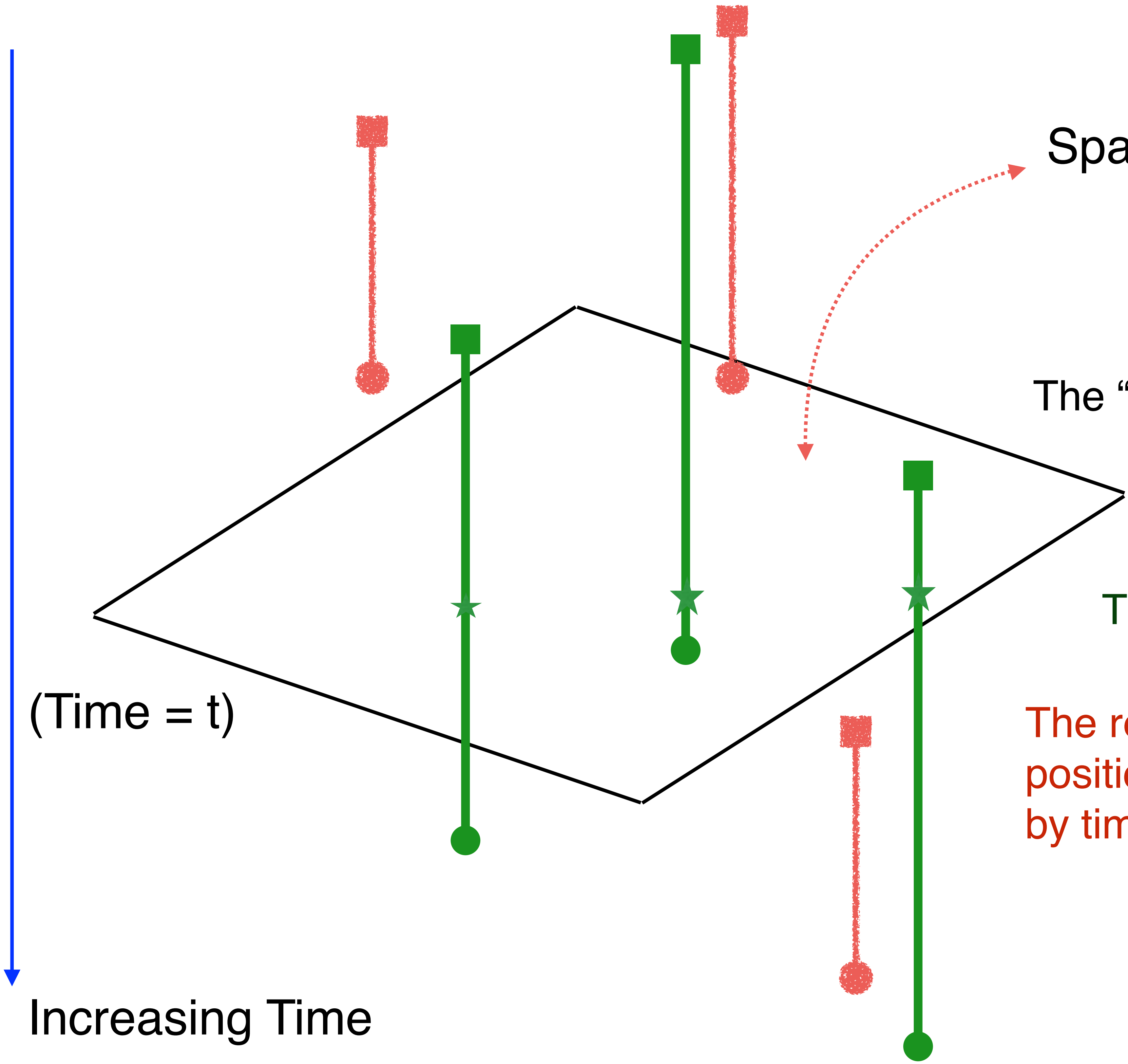
The rate function $R(x,\phi) = C \log_2 \left(1 + \frac{1}{N_0 + \sum_{y \in \phi_t \setminus \{x\}} l(||y-x||)} \right)$



$$\phi_t = \{x_1, \cdots, x_{N_t}\} \ , \ x_i \in \mathbf{S}$$

(Configuration at time t).

SBD Model - Special Case



The "■" appear as a PPP on $\mathbf{S} \times \mathbb{R}$

The green ★ represent ϕ_t

The red lines represent space-time position of points that are either dead by time t or not yet born at time t.

Natural Questions to ask on the Model

- When is ϕ_t Ergodic ? (i.e. admit an unique stationary regime)

  This has design implications for example in determining how much traffic to off-load from cellular to D2D.

  In particular, is a phase-transition for finite mean delay.

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  In particular, is a phase-transition for finite mean delay.

- When ϕ_t is ergodic, can one say something about the steady-state point process ?

  Formulas for mean delay and intensity in steady state.

Main Result - Ergodicity Criterion

Theorem -

Denote by $\lambda_c = \frac{Cl(T)}{\ln(2) L \int_{x \in \mathbf{S}} l(||x||) dx}$. Then,

(1) $\lambda > \lambda_c \implies \phi_t$ admits no stationary regime. (a.k.a. stable)

(2) Under further assumptions that $r \to l(r)$ is bounded and monotone,

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Corollary:

ϕ_t is always unstable for the popular power law path-loss function

$l(r) = r^{-\alpha}$ for all $\alpha > 2$ since $\int_{x\in\mathbf{S}} ||x||^{-\alpha}dx = \infty$

Intuition for Phase Transition

Assume ϕ_0 is the steady-state point process on $\mathbf{S}$ with intensity β for the dynamics to guess the phase-transition point.

Rate-Conservation - "On average, what comes in is what goes out"

Total speed at which bits arrive	$\lambda|\mathbf{S}|L = \mathbb{E}\left[\sum_{x \in \phi_0} R(x, \phi_0)\right]$	***Total speed at which bits depart.***

Using the definition of Spatial Palm probability, the above simplifies to

$$\lambda L = \beta C \mathbb{E}^0_{\phi_0}\left[\log_2\left(1 + \frac{l(T)}{N_0 + I(0, \phi_0)}\right)\right]$$

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Under the assumption, concentration of interference holds, i.e.

$$\sum_{y\in\phi_0} l(||y||) \approx \mathbb{E}[\sum_{y\in\phi_0} l(||y||)] = \beta \int_{y\in\mathbf{S}} l(||x||)dx$$

Thus (1) simplifies to give $\lambda L = \beta C \log_2\left(1 + \frac{l(T)}{N_0 + \beta\int_{x\in\mathbf{S}} l(||x||)dx}\right) = f(\beta)$

Intuition for Phase Transition

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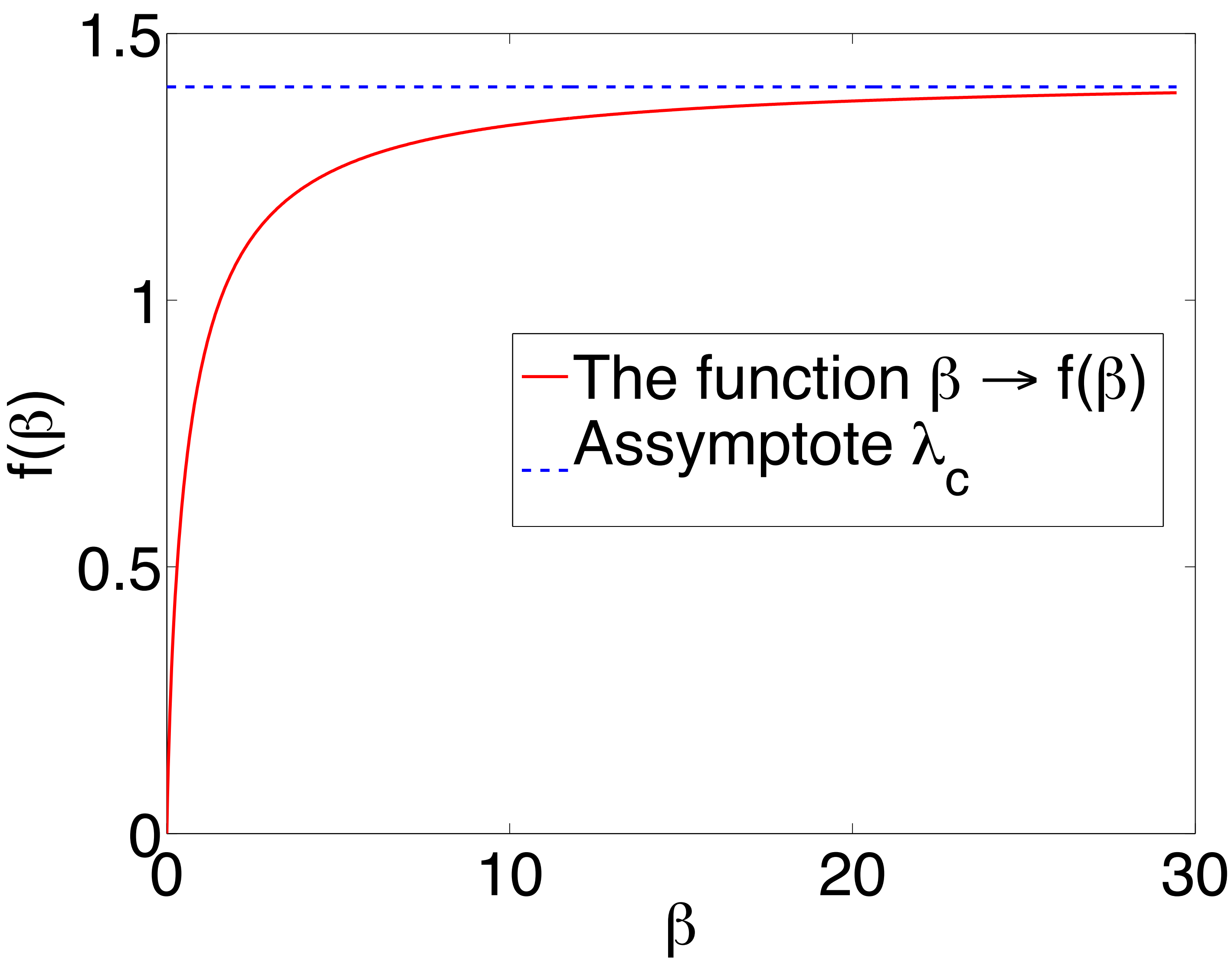
The rate-conservation can be simplified to the following.

$$\lambda L = \beta C \log_2 \left(1 + \frac{l(T)}{N_0 + \beta \int_{x \in \mathbf{S}} l(||x||) dx} \right) = f(\beta)$$



We need $\lambda < \lambda_c$ for the equation $\lambda L = f(\beta)$ to hold.

Clustering in Steady State

Theorem :

Let $B(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ be any non-increasing function. Then

$$\mathbb{E}^0_{\phi_0}\left[\sum_{y \in \phi_0^{Tx} \setminus \{Tx(0)\}} B(||y||)\right] \geq \mathbb{E}\left[\sum_{y \in \phi_0^{Tx}} B(||y||)\right]$$

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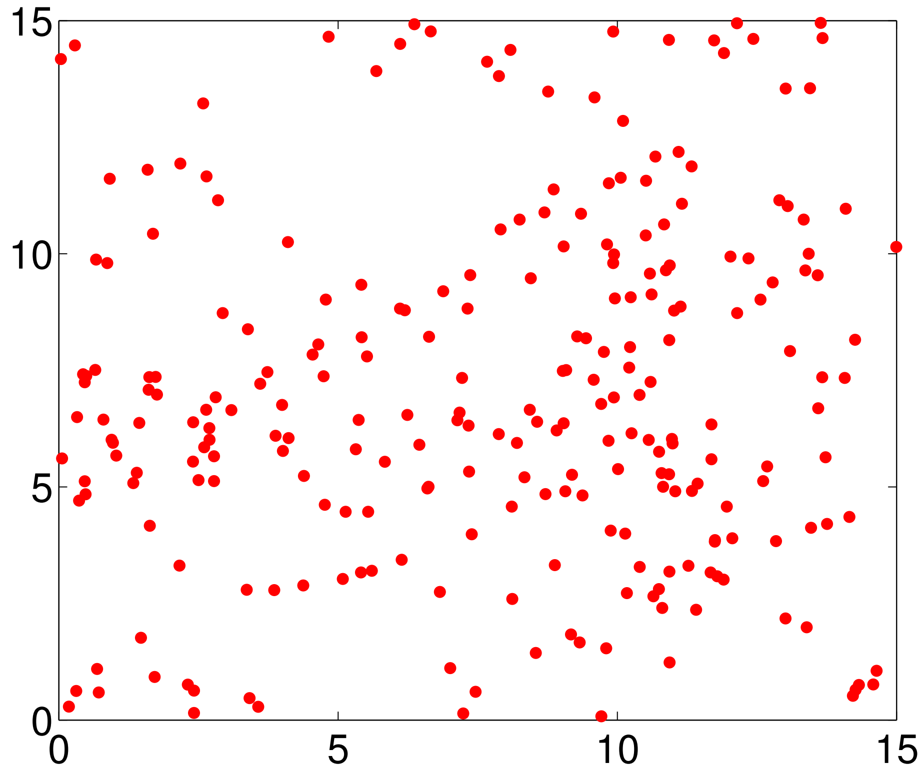
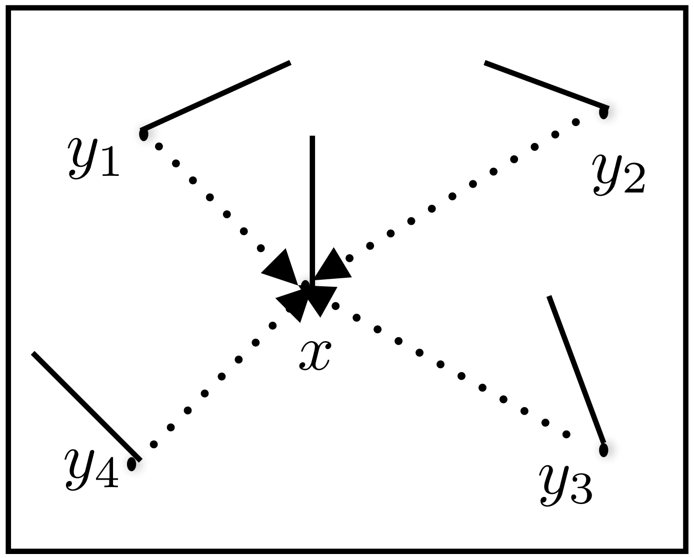


This clustering invalidates the Poisson assumption, but indicates, the Poisson approximation can be a bound.

An Understanding of Clustering

A point in a crowded region of space is slowed down and in turn slows down others near it.



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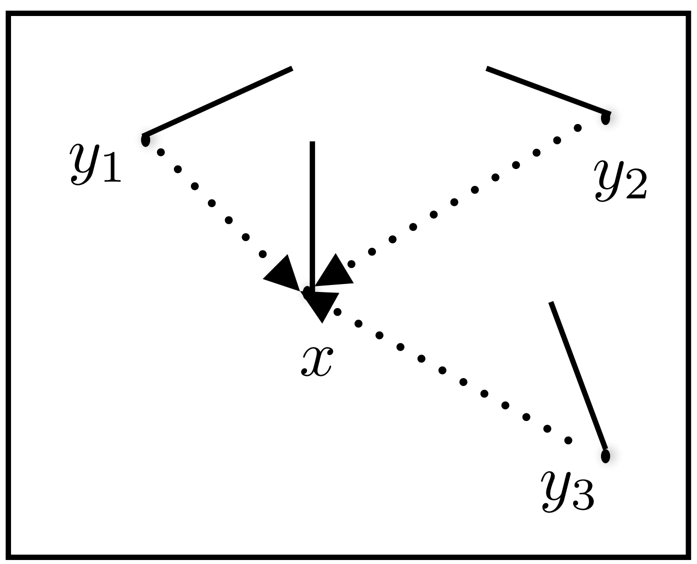

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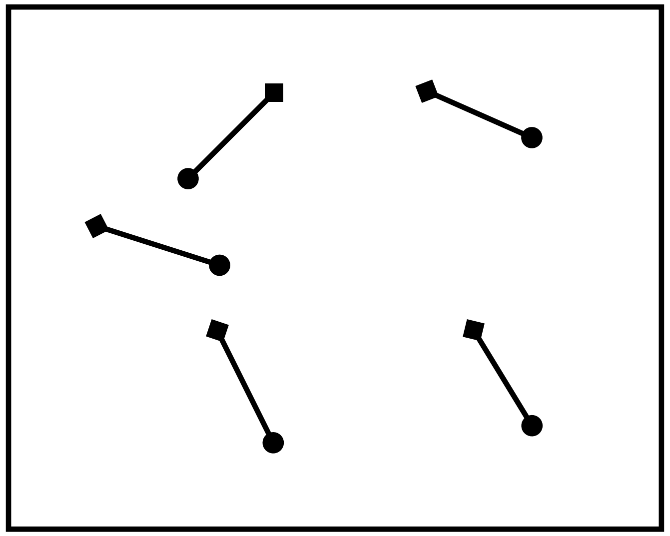
Intuitively, expect some form of clustering in steady state which the theorem formalizes.

Formulas for Mean number of links



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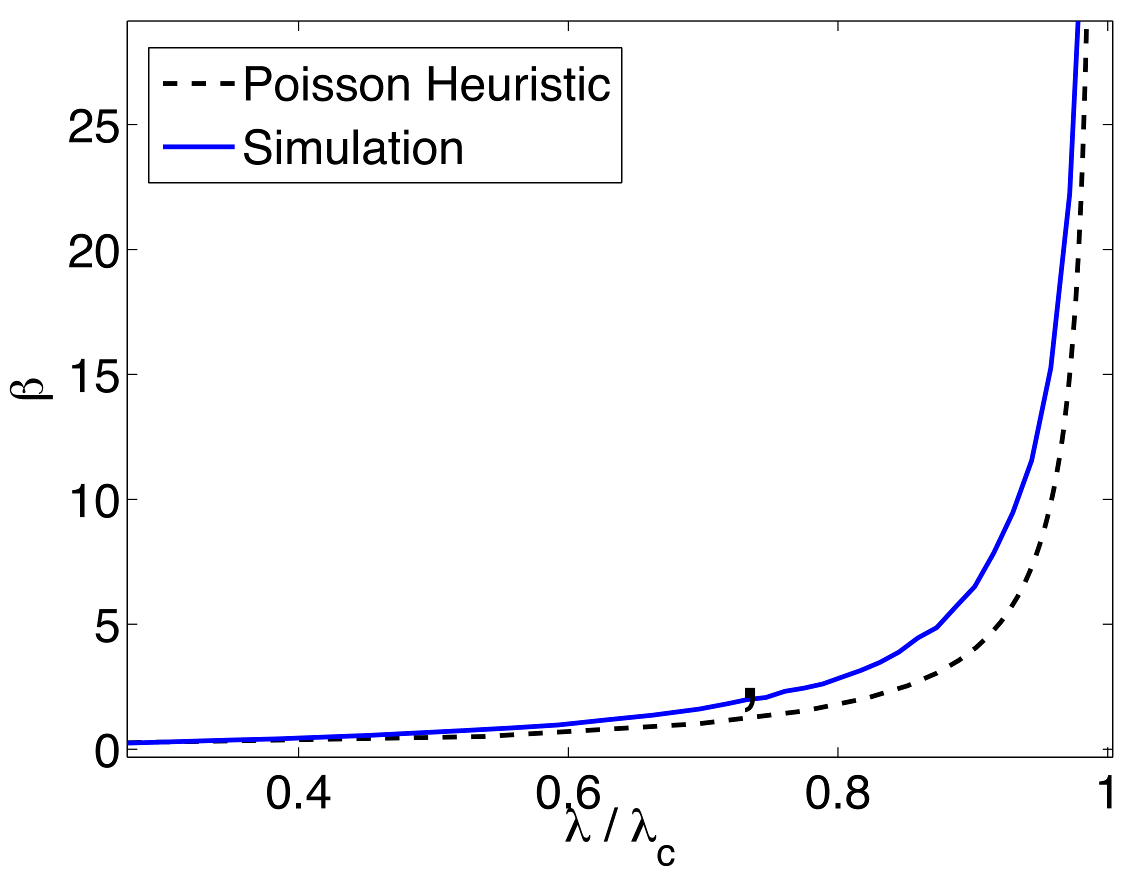
Steady State Formulas - Poisson Heuristic

The Poisson approximation gives a simple heuristic for computing β

$$\lambda L = \beta \mathbb{E}\left[\log_2\left(1 + \frac{1}{N_0 + \sum_{y \in \phi_0} l(||y||)}\right)\right]$$

$$= \frac{\beta_f}{\ln(2)} \int_{z=0}^{\infty} \frac{e^{-N_0 z}(1 - e^{-z})}{z} e^{-\beta_f \int_{x \in \mathbf{S}} (1 - e^{-z l(||x||)}) dx} dz$$

The largest solution to the above fixed point equation gives a heuristic formula



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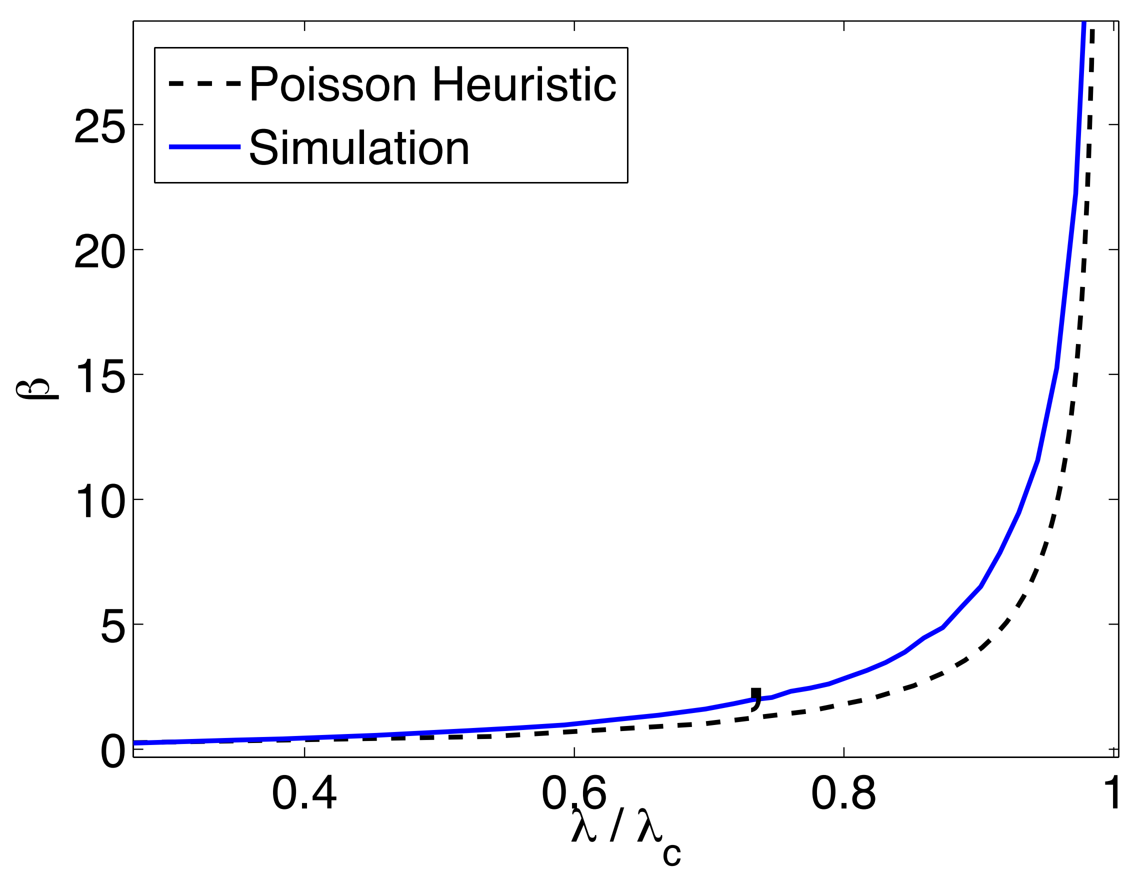
The largest solution to the above fixed point equation gives a heuristic formula



As expected, performs poorly.
However, we conjecture that

Steady State Formulas - Second Order Heuristic

A heuristic that accounts for correlation i.e. clustering

The Approximation

1. Any single tagged particle interacts with a static non-random environment $\hat{I}$

$$\beta_s = \frac{\lambda L}{C \log_2 \left(1 + \frac{1}{N_0 + \hat{I}}\right)}$$ *(Similar to the Poisson Approximation)*

2. Pairs of points are not independent *(Accounting for the Clustering)*

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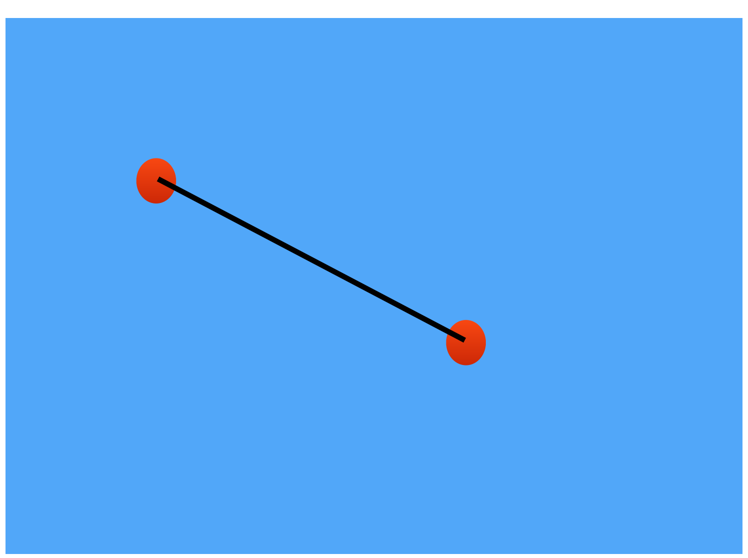


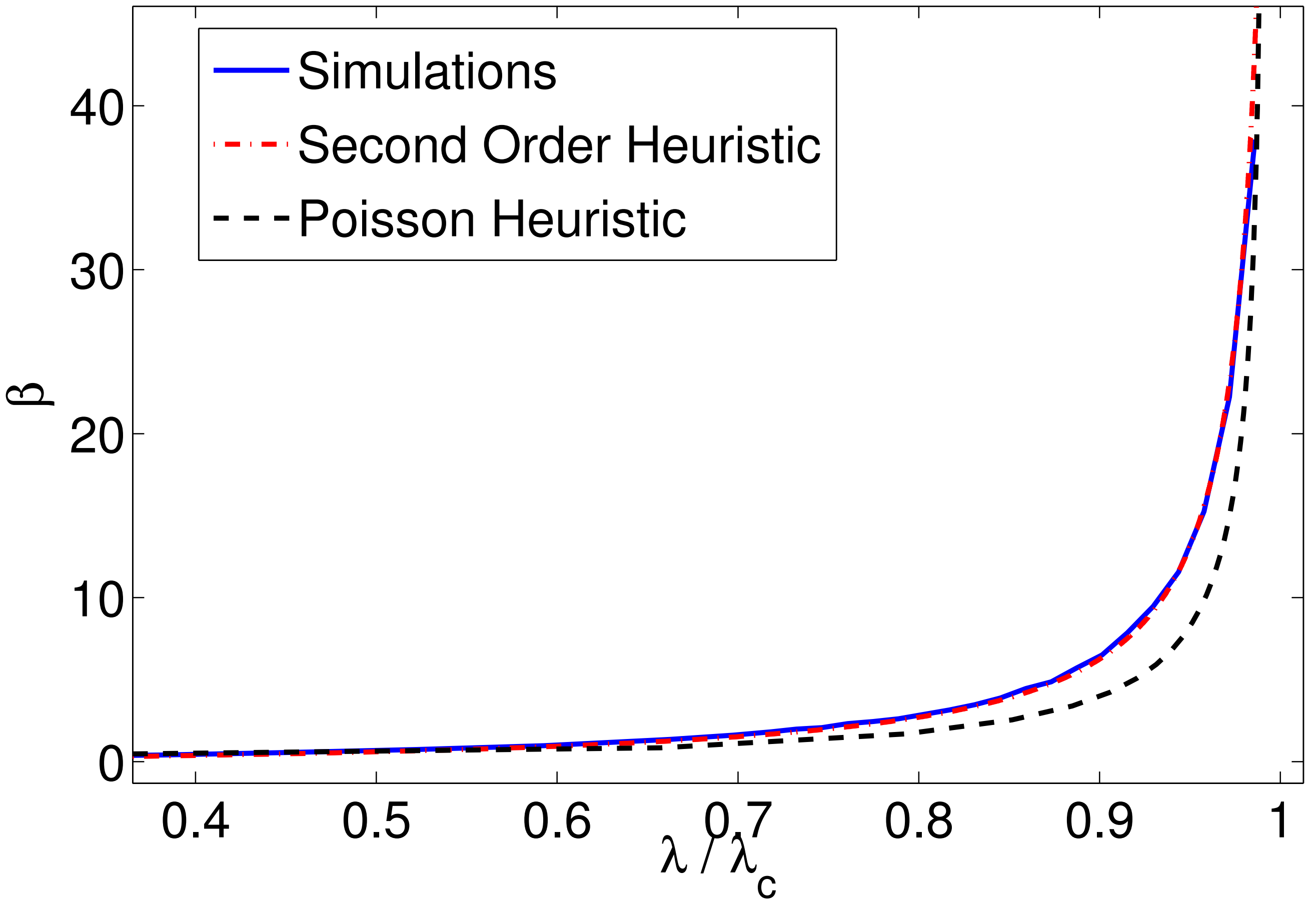
Pairs of particles interact with a environment. and with each other

Steady State Formulas - Second Order Heuristic

$$\beta_s = \frac{\lambda L}{C \log_2\left(1 + \frac{1}{N_0 + I_s}\right)}$$

where I_s is the smallest solution to the fixed point equation

$$I_s = \lambda L \int_{x \in \mathbf{S}} \frac{l(||x||)}{C \log_2\left(1 + \frac{1}{N_0 + I_s + l(||x||)}\right)} dx$$

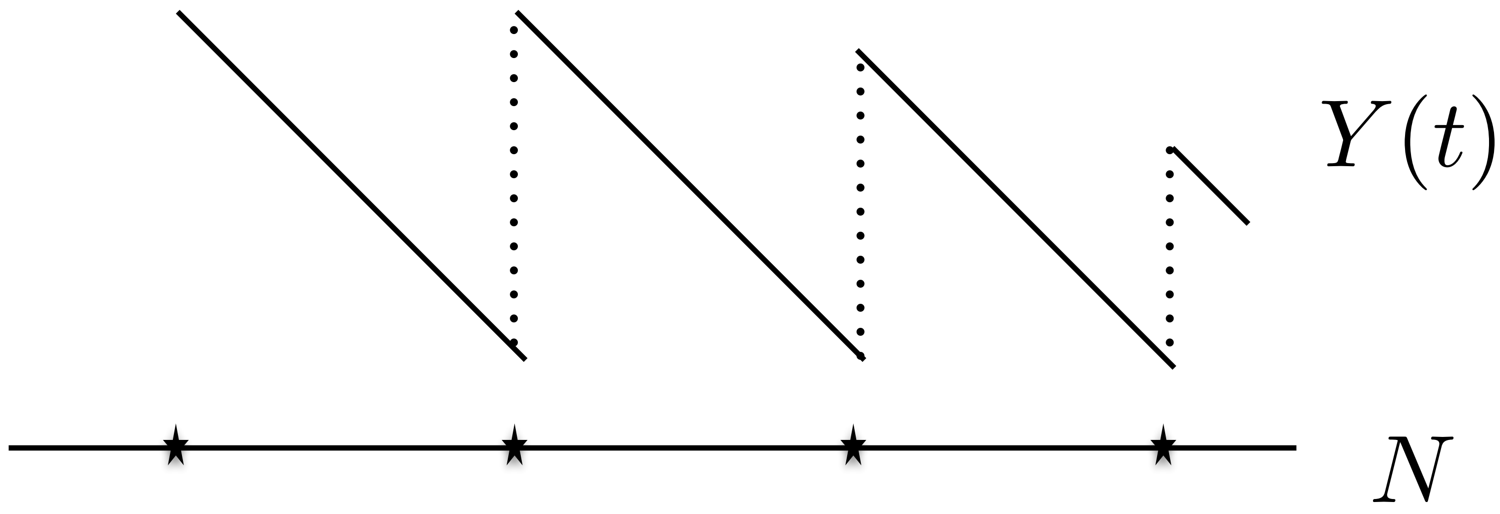


The second-order heuristic performs much better than the Poisson heuristic as it accounts for clustering.

Proof sketch for Stability Phase Transition

Proof Idea - Necessary Condition

Assume stability and write down ‘Rate Conservation Equations’.
Then find a contradiction.



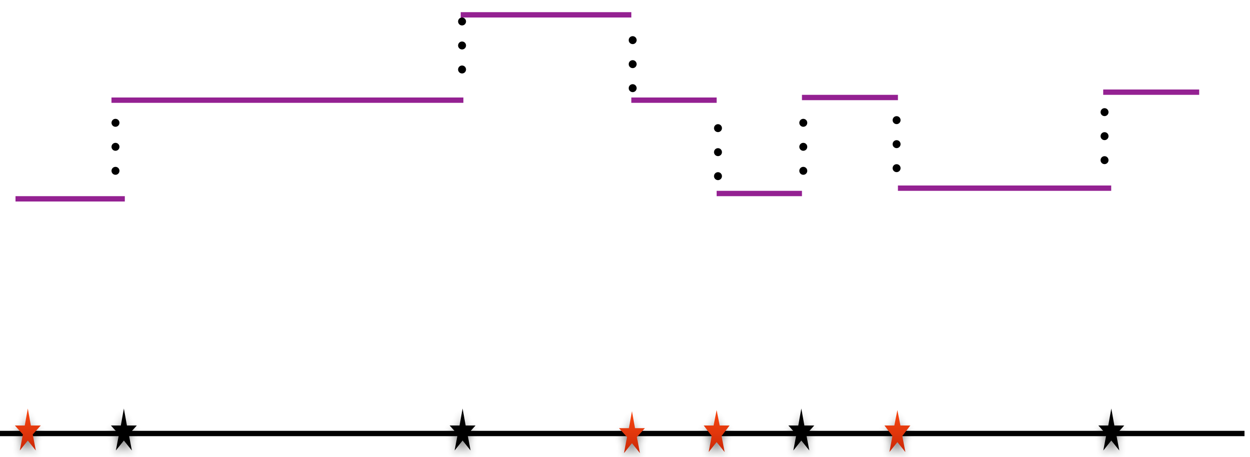
Both process are stationary

If $$Y(t) = Y(0) + \int_{s=0}^{t} D(s)ds + \int_{s=0}^{t} (Y(s) - Y(s^-))N(ds)$$

Implies $$\mathbb{E}[D(0)] + \lambda_N \mathbb{E}_N^0 [Y(0) - Y(0^-)] = 0$$

Proof Idea - Necessary Condition

$\phi_t(\mathbf{S})$

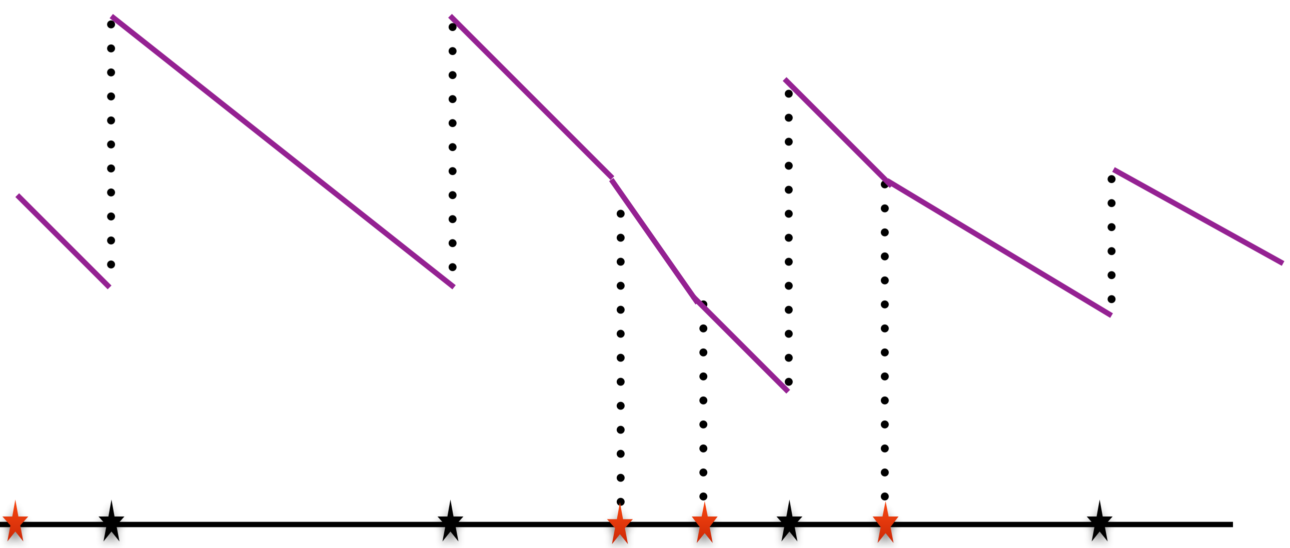


Red - Epochs of Death
Black - Epochs of Arrivals

RCL implies $\lambda |S| = \lambda_d$

(1)

Total bits left in the network i.e. remaining ‘workload’

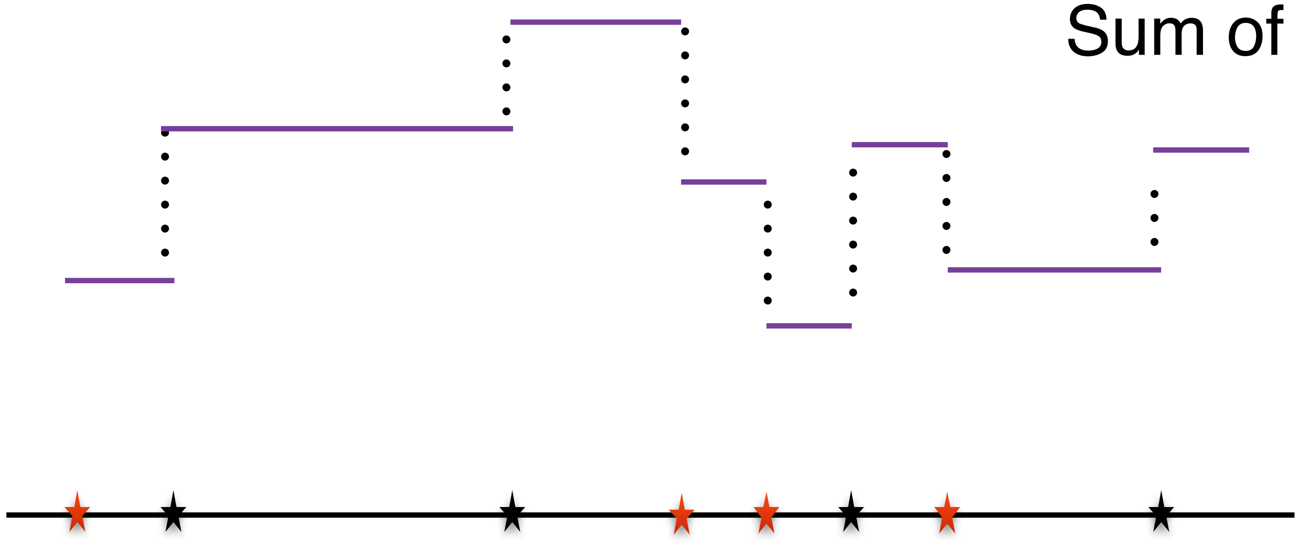


RCL implies

$$\lambda |S| L = \mathbb{E}\left[\sum_{x \in \phi_0} R(x, \phi_0)\right]$$

(2)

Proof Idea - Necessary Condition



Sum of interference seen at all points

$$Y(t) = \sum_{x \in \phi_t} I(x, \phi_t)$$

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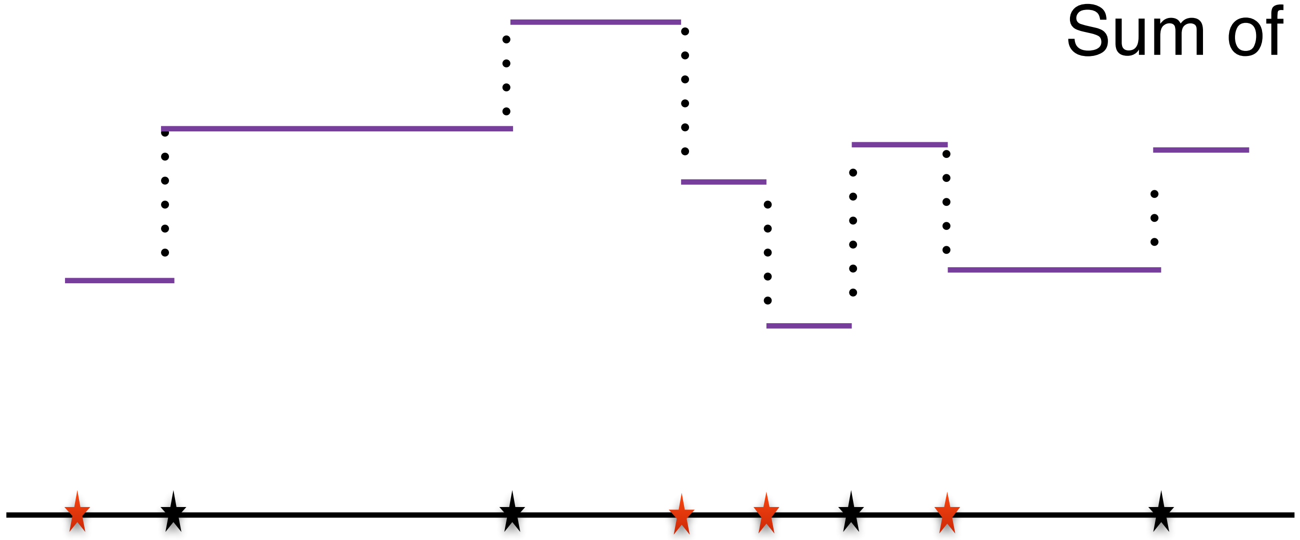
Proof Idea - Necessary Condition



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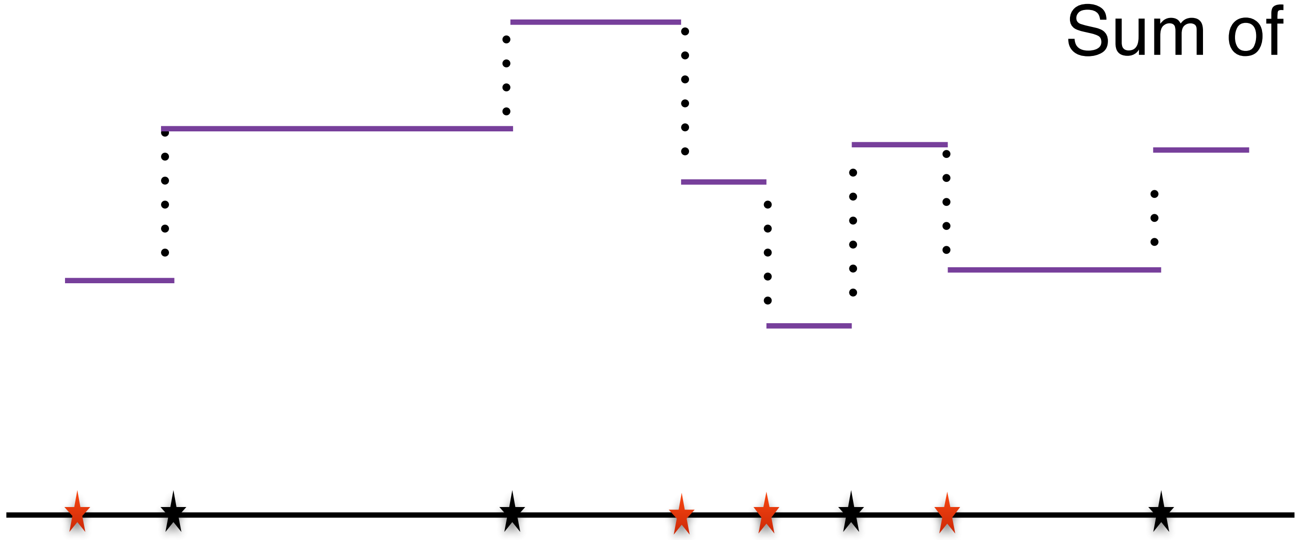

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$$\mathbb{E}[\mathcal{I}] = 2\frac{\mathbb{E}[\phi_0(\mathbf{S})]}{|S|}\int_{x \in \mathbf{S}} l(||x||)dx$$

Linearity of Expectation

Handle this measure through ***Papangelou's Theorem***

Proof Idea - Necessary Condition

We have the following 3 rate conservation equations

$$\lambda|S| = \lambda_d \qquad (1)$$

$$\lambda|S|L = \mathbb{E}\left[\sum_{x\in\phi_0} R(x,\phi_0)\right] \qquad (2)$$

$$\mathbb{E}^0_b[\mathcal{I}] = \mathbb{E}^0_D[\mathcal{D}] \qquad (3)$$

The Death Point process admits as stochastic intensity - $\mathbf{R}_t = \sum_{x\in\phi_t} R(x,\phi_t)$

with respect to the filtration $\mathcal{F}_t = \sigma(\phi_s : s \le t)$

Papangelou's theorem implies $\left.\frac{d\mathbb{P}^0_d}{d\mathbb{P}}\right|_{\mathcal{F}_{0^-}} = \frac{\mathbf{R}_0}{\mathbb{E}[\mathbf{R}_0]}$ *(Structure in the Dynamics)*

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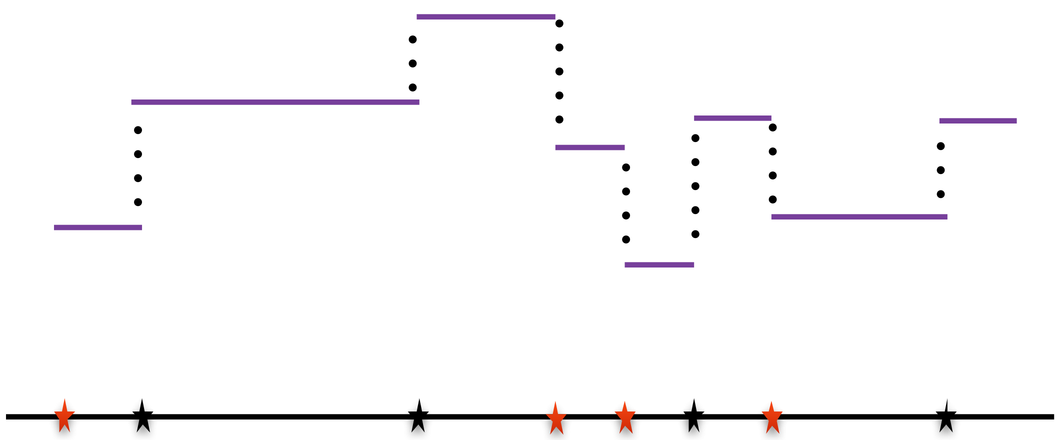
- On simplifying, can see that equations (1), (2), (3) and the relation $\lambda > \lambda_c$ can't hold simultaneously.

  Thus $\lambda > \lambda_c \implies \phi_t$ admits no stationary regime. ■

Proof Idea - Clustering



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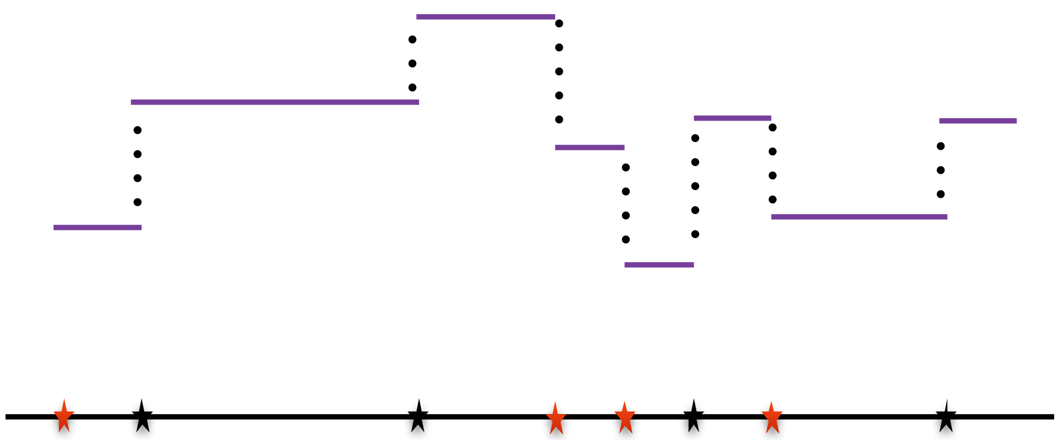

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$$\mathbb{E}_b^0[\mathcal{I}] = \mathbb{E}_D^0[\mathcal{D}]$$

$$\mathbb{E}[I(0, \phi_0)] = \frac{\beta}{\lambda L} \mathbb{E}_{\phi_0}^0[R(0, \phi_0) I(0, \phi_0)]$$

$$\leq \mathbb{E}_{\phi_0}^0[R(0, \phi_0)] \mathbb{E}_{\phi_0}^0[I(0, \phi_0)]$$

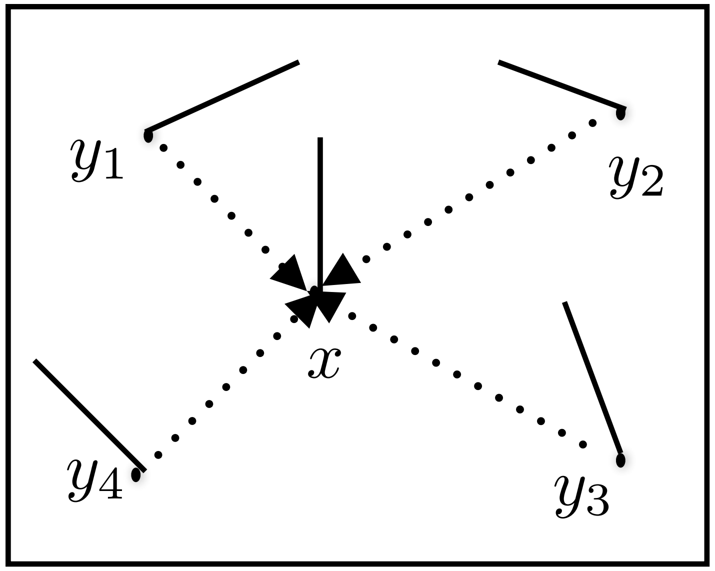
Since $R(0, \phi_0)$ *is a deterministic non-increasing function of* $I(0, \phi_0)$

Rearranging the terms further gives the result. ■

Proof Idea - Sufficient Condition

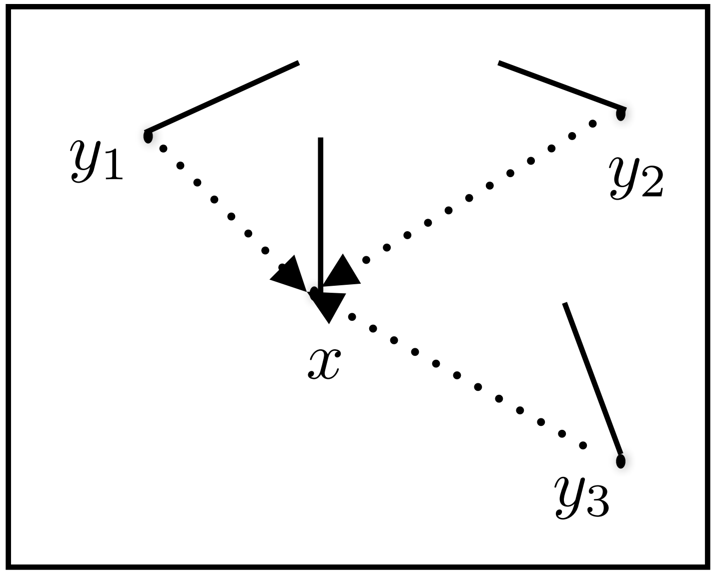
The dynamics has this inherent ‘subset’ monotonicity.

Thus, can study a certain ϵ approximation such that $R_\epsilon(x,\phi) \leq R(x,\phi)$



$$\phi_1 = \{(x, Tx(x)), (y_1, Tx(y_1)), (y_2, Tx(y_2)), (y_3, Tx(y_3)), (y_4, Tx(y_4))\}$$

$$R(x,\phi_1) \geq R(x,\phi_2)$$



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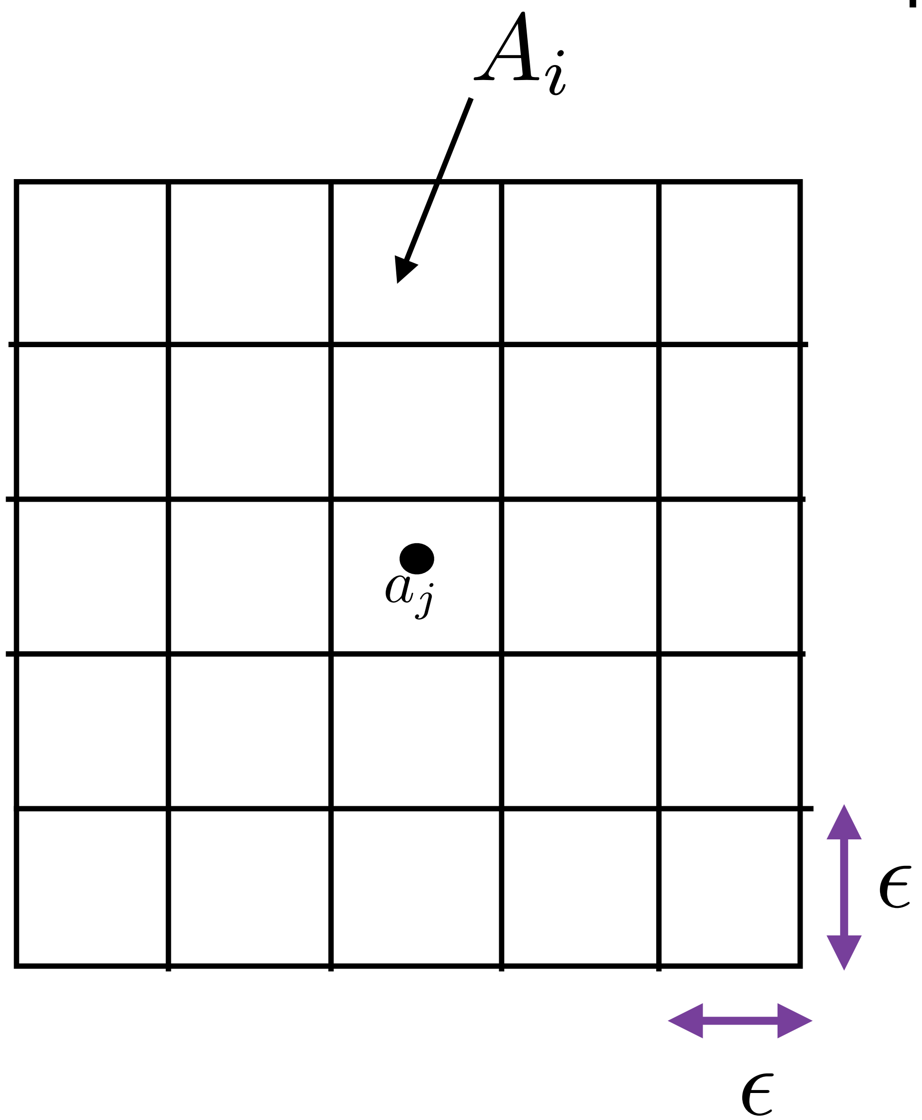
Proof Idea - Sufficient Condition

The dynamics has this inherent ‘subset’ monotonicity.

We construct a discrete upper bound dynamics by tessellating the space $\mathbf{S}$



Denote by ϕ_t^ϵ as the configuration at time t in this ϵ approximate system

Let $\mathbf{X}(t) \in \mathbb{N}^{N_\epsilon}$ where $X_i(t) = \phi_t^\epsilon(A_i)$.

Want $\mathbf{X}(t)$ as a Markov Chain on $\mathbb{N}^{N_\epsilon}$
and want to work out a natural coupling with ϕ_t

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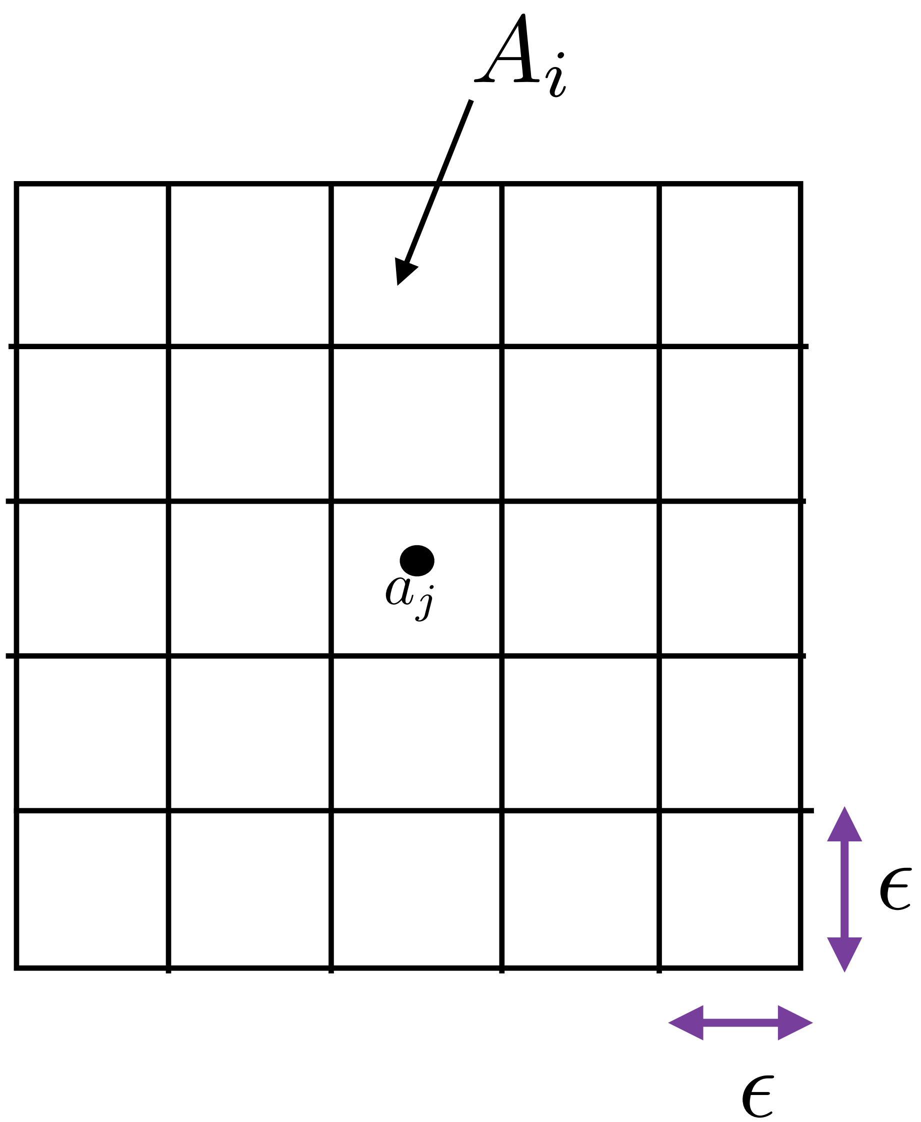

1. Arrivals - PPP on $\mathbb{R} \times \mathbf{S}$ with intensity λ
2. IID Exponential File Sizes of mean L.
3. $l_\epsilon(x, y)$ - The path-loss function is such that $l_\epsilon(x, y) = l(a_i, a_j)$ for all $x \in A_i$ $y \in A_j$

$\Longrightarrow$ $\mathbf{X}(t)$ ***is a Markov Chain***

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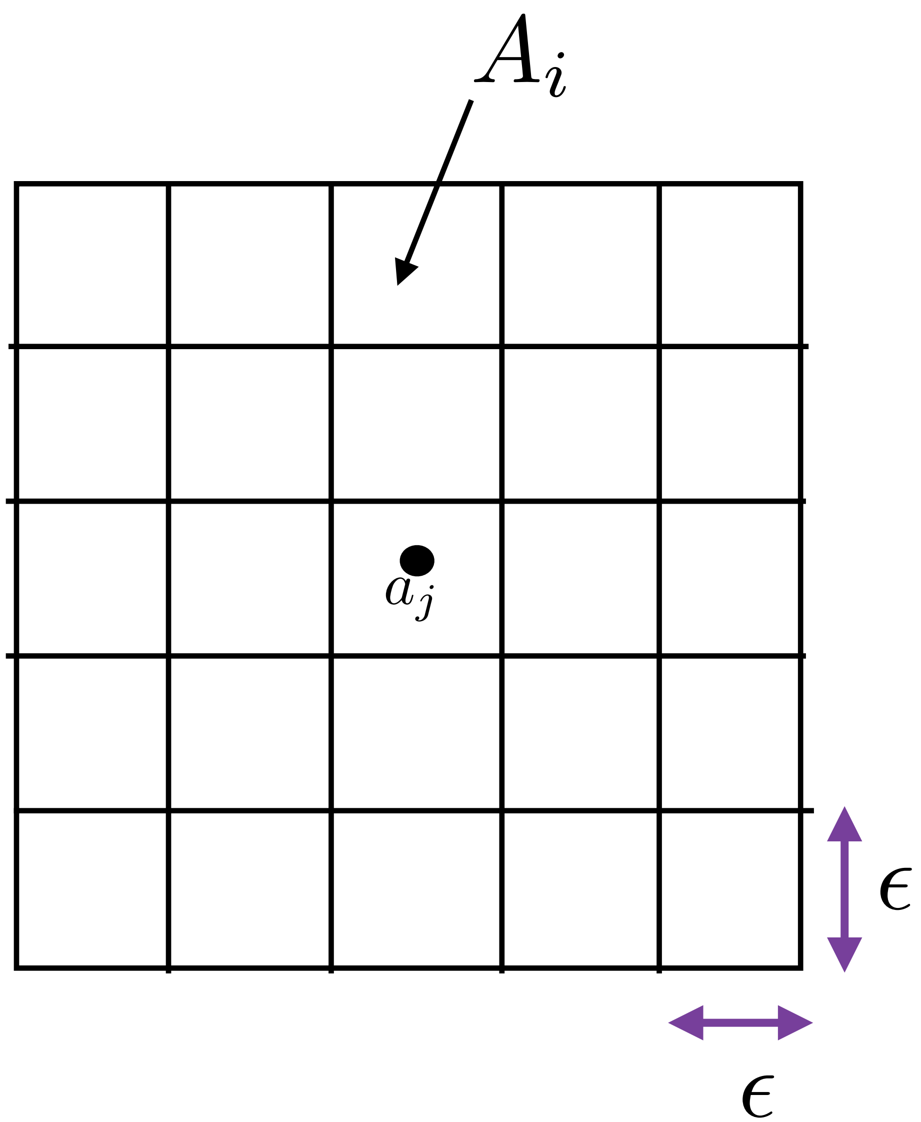

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$\implies \mathbf{X}(t)$ ***is a Markov Chain***

Subset Monotonicity further gives that if $l_\epsilon(x,y) \geq l(x,y) \forall x, y \in \mathbf{S}$ then,

$X(t) \succcurlyeq (\phi_t(A_i))_{i=1}^{N_\epsilon}$. Furthermore,

$$\mathbb{P}[\lim_{t\to\infty} ||X(t)||_1 < \infty] \leq \mathbb{P}[\lim_{t\to\infty} \phi_t(\mathbf{S}) < \infty]$$

Proof Idea - Sufficient Condition

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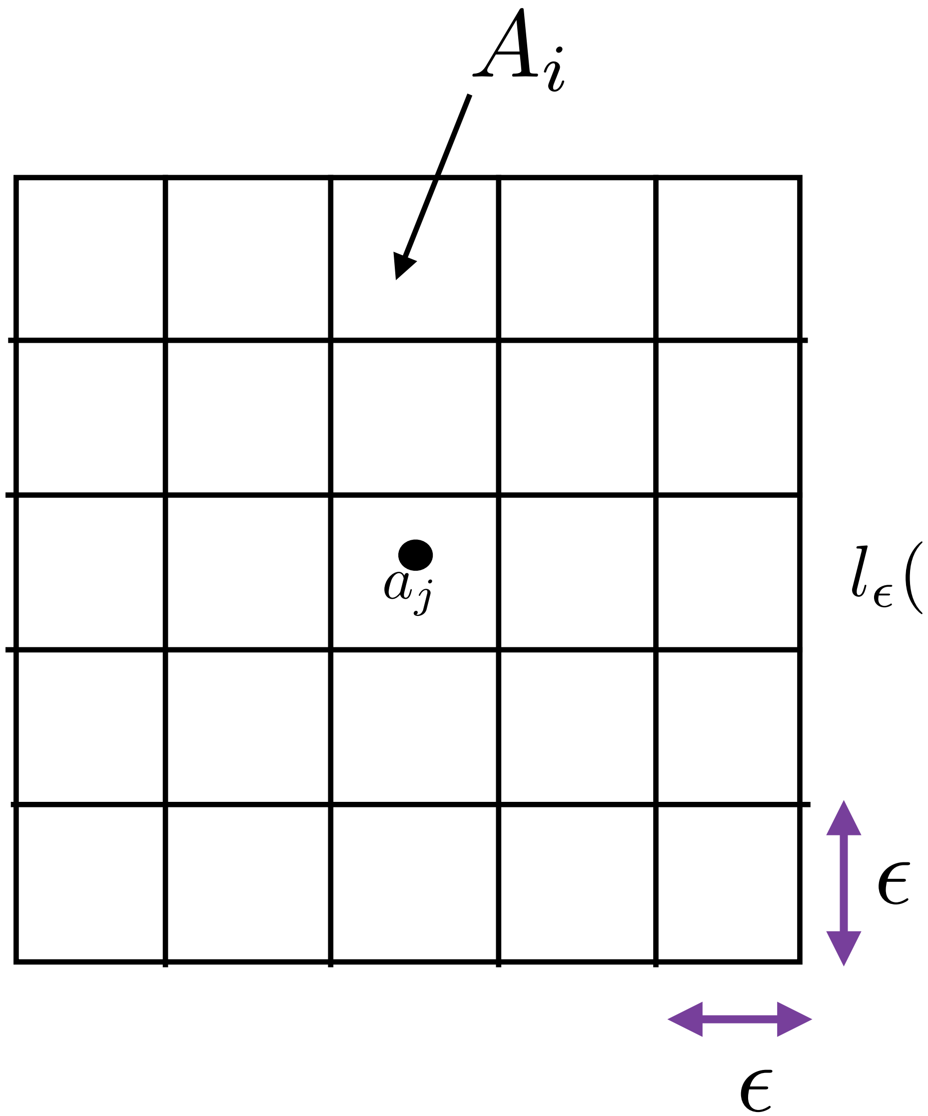

Need to define a path-loss function so that

- $l_\epsilon(x,y) = l(a_i, a_j)$ for all $x \in A_i \;\; y \in A_j$
- $l_\epsilon(x,y) \geq l(x,y) \forall x, y \in \mathbf{S}$

$$l_\epsilon(a_i, a_j) = \sup\{l(||b_i - b_j|| : ||a_i - b_i||, ||a_j - b_j|| \in \{0, \epsilon\}\}$$

Defines a discrete upper bound dynamics such that

$$\mathbb{P}[\lim_{t\to\infty} ||X(t)||_1 < \infty] \leq \mathbb{P}[\lim_{t\to\infty} \phi_t(\mathbf{S}) < \infty]$$

$X(t)$ ***stable*** $\Longrightarrow$ ϕ_t ***has an unique stationary regime***

Proof Idea - Sufficient Condition

The Evolution

$X_i \to X_i + 1$ at rate $\lambda \epsilon^2$

$X_i \to X_i - 1$ at rate $\frac{1}{L} C X_i \log_2 \left(1 + \frac{1}{N_0 + I_i^\epsilon(X)}\right)$

$$I_i^\epsilon(X) = \sum_{j=1}^{N_\epsilon} (X_j - \mathbf{1}(j = i)) l_\epsilon(a_i, a_j)$$

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Fluid Scale Evolution.

Analyze this evolution through Fluid Limit techniques of [Dai 95] [Massoulié, 07].

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By letting $\epsilon \to 0$, we can conclude that
$\lambda < \lambda_c \implies \phi_t$ admits an unique stationary regime.

■

Conclusions and Future Questions

- Framework to account for spatial-temporal interactions in a wireless network.
  - Stability Criterion
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  Dynamic Capacity - Maximal Arrival rate over all scheduling ?

- Proof techniques for the infinite plane system ?

  We believe the phase-transition value is the same, but no proof.

Thank You very much for your time.