# Spatial Stochastic Models in Wireless and Data Networks

by

**Abishek Sankararaman, B.S., M.S.**

**Ph.D. Qualifying Proposal**

Decision, Information and Communications Engineering (DICE)

Department of Electrical and Computer Engineering

The University of Texas at Austin

**Committee Members**

Prof. François Baccelli (supervisor)

Prof. Sanjay Shakkottai

Prof. Gustavo de Veciana

Prof. Joe Neeman

Prof. Alexandros.G. Dimakis

THE UNIVERSITY OF TEXAS AT AUSTIN

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Supervisor: Prof. François Baccelli

In this proposal, we study various spatial stochastic models of networks coming from applications in wireless communications and social sciences. In all these applications, we propose suitable stochastic processes to model and capture the phenomena and metrics of interest. We use the models to derive design insights and algorithmic ideas for the operation of these networks.

In the first part, we study a problem of space-time particle dynamics motivated by the spectrum-sharing dynamics of Device-to-Device ad-hoc wireless networks. In this project, we propose an interacting particle system where the particles model the wireless links and the 'interactions' between the links capture the effects of interference and spectrum sharing which are modeled by considering the 'Interference as Noise' paradigm. Roughly speaking points arrive in space (a compact subset of $\mathbb{R}^2$), stay for a duration governed by the local configuration of points, and then exit after completion of a file transfer. We study this particle dynamics to establish a sharp phase-transition for when the dynamics admits a stationary regime. Moreover, we show that when the dynamics is stationary, the steady state exhibits a form of statistical clustering. We leverage these results to propose bounds, heuristics and assymptotics for the key performance metrics such as delay of a typical link and the mean number of links in the network in steady state. We then propose a future work wherein we would want to extend the above dynamics to the entire Euclidean space $\mathbb{R}^2$ instead of just restricting to a compact set. This extension we believe to be interesting

both from a mathematical and in terms of network scalability point of view as detailed in the sequel.

In the second part, we consider a problem of base station association motivated by recent trends in cellular systems such as the 'Google-Fi' project and the MVNO (Mobile Virtual Network Operators ). We pose the association problem as a stochastic optimization problem, where the mobile phone will pick a base-station to associate with depending on the instantaneous 'information' about the network it has by solving an appropriate optimization problem. We give a formal framework to capture information at a mobile phone through appropriate filtrations of a probability space and give simple characterizations of the optimal association policy. We also propose certain 'data-dependent' policies that are strictly sub-optimal, but are more practical to implement in the sense they are non-parametric. We exhibit a simple policy that requires the mobile phone to learn very little about the instantaneous network, and which is still nonetheless optimal under a certain asymptotic regime. We further show through simulations that for practically reasonable values of the parameter, the heuristic performs almost as-well as the optimal policy thus making this an attractive choice in practice.

In the third part, we propose an ongoing work wherein we study the problem of community-detection on a spatial random graph. Community detection or clustering refers to the task of partitioning an underlying population into groups so that members are similar in the same group and are dissimilar across groups. In this thesis, we consider a particular framework wherein the underlying population is composed of points of a point-process (or more generally have an embedding in a metric space) and the group membership information is encoded by a noisy graph process on the points. Mathematically, we propose a natural extension of the classical Stochastic Block Model to the spatial graph case. We give some first results on this model and expand on future work we intend to pursue for the thesis.

# Table of Contents

# Chapter 1

# Introduction

The goal of this report is to propose and mathematically analyze certain spatial stochastic models for problems in wireless and social networks. The main applications this thesis will focus on are Device-to-Device wireless networks, multi-operator cellular networks, and Social Networks with latent variables. We will view these different applications through a common lens of stochastic geometry which is a very powerful modeling tool. Although these topics have been studied in the research community for a long time, new network paradigms bring new challenges in the design of these systems. This thesis is motivated by these new developments, for which we propose tractable stochastic models which in turn lead to design insight and new algorithmic ideas for operating these networks.

The main technical tool we use in this thesis is the homogeneous Poisson Point Process (PPP) on Euclidean Space which is defined formally in the sequel. It although appears to be simplistic, is still rich and useful to model different problems and to capture various phenomena. From a mathematical point of view, the PPP is part of a broader class of study known as Stochastic Geometry, which is a well studied sub-field of both theoretical and applied probability. In recent years, the tools from Stochastic Geometry have become very prevalent in the design and analysis of wireless networks. Indeed the books of [1], [2] are now a standard reference for stochastic models in wireless network analysis. In this thesis, we contribute to this theme by proposing a tractable dynamic version of some of the stochastic models discussed in [1] and [2] for wireless networks. This dynamics is mathematically new and provides a new understanding of stochastic stability and delay in a large wireless network.

(This will be made precise in the sequel). In the second part, we use tools from Stochastic Geometry to model a cellular network with multiple operators or technologies (3G,4G etc). We show that, by cleverly exploiting the presence of multiple cellular networks operating on non-overlapping spectrum, a typical mobile phone will experience a gain in performance which we coin 'Technology Diversity'. We leverage simple ideas from stochastic geometry to propose practical base-station association schemes in this setting. In the third part, we use the tools of Stochastic Geometry to develop a new model of community formation in social networks. Our model is based on the broad class of Latent Space Social models ( [3], [4]) and mathematically is a natural extension of the classical Stochastic Block Model ( [5]) to the spatial setting.

This report is organized as follows. In Chapters 2 and 4, we discuss already finished and published work. In Chapters 3 and 5, we discuss ongoing and future work. In Chapter 6, we list the coursework undertaken and in Chapter 7, we list the publications which are a part of the thesis.

## 1.1 Summary and the main contributions in each chapter

In this section we highlight the main technical contributions in each chapter.

### 1.1.1 Chapter 2 - Spatial Birth-Death Wireless Networks

In this chapter, we focus on a problem of spatio-temporal spectrum sharing dynamics in D2D networks. D2D networks are a form of ad-hoc wireless networks that operate without the presence of a centralized controller or a fixed communication architecture. Although ad-hoc wireless networks have been considered for long, they are being looked with renewed interest as they are considered a key technology in the upcoming 5G standards. In this proposal, we study how to operate a D2D network to keep the network dynamically 'stable'.

Mathematically, we propose a new stochastic model of spatial birth-death process that are not Gibbs, but are based on and Information-Theoretic form of interactions. Roughly speaking, our model is a form of interacting particle system, where particles (or more generally wireless links) arrive in time in a compact set of $\mathbb{R}^2$, and stay for a duration governed by the local configuration of active links and then exit the network after a file transfer. This is a natural model of a D2D network, whereby users switch on their requests randomly in space and time sharing a common spectrum. We analyze this particle dynamics to derive an exact phase-transition criterion for stochastic stability. We find a critical rate of arrival, below which the network is stable and above which the network is unstable. We further show that when this particle dynamics is stable, the steady-state locations of particles exhibit a form of statistical clustering, which we further leverage to propose bounds, heuristics and asymptotic on network performance characteristics such as delay and mean number of links per unit space in the stationary regime. The contents of this chapter appears in [6].

Our particle interaction system is a new form of queuing network. In our model, the spectrum that is available distributed in space can be thought of as a 'server', providing service to the links. However, we show that any simplistic representation of our model as an equivalent queue or an equivalent static spatial system will predict delays that are significantly off from the one obtained by our model. We show that the clustering phenomena we prove is very crucial in understanding key performance metrics of such networks and any simplifying model neglecting clustering will lead to poor estimates. From an information-theoretic point of view, one can interpret our result and phase-transition as a form of dynamic network capacity for a collection of additive noise Gaussian point-to-point channels interacting through interference which is treated as noise. We believe that this viewpoint can be generalized in the future where one can potentially consider a dynamics of multiple access or broadcast channels interacting in space and consider the associated phase-transition and the dynamic network capacity. From an application perspective, we give the first known expressions for mean delay of a typical link in such spatial wireless dynamics. These simple expressions can shed light on designing D2D networks guaranteeing a certain Quality of Service to the users.

### 1.1.2 Chapter 3 - Interacting Queues on $\mathbb{Z}^d$

In Chapter 3, we propose an ongoing work wherein we extend the above model to the infinite plane $\mathbb{R}^d$ instead of confining the links to a compact set of Euclidean space. Although this seems incremental and technical, we believe that the model is fundamentally different from the model considered in Chapter 2. For one, the mathematical analysis and proof techniques need to be completely different, which will possibly lead to the development of newer mathematical arguments, which are interesting in their own right. From a practical perspective, the model on the infinite plane has a potential to capture a lot of phenomena which cannot be studied in the finite model. For example, the infinite model has a potential to exhibit multiple stationary solutions depending on the initial conditions. Such mathematical phenomena pertain to long-range correlations and imply a form of long-term unfairness in the network. Thus, it will be desirable to not operate the networks in these regimes even though the network is stable. The infinite model of a network is also the right framework to assess *scalability* of the system. Scalability in this scenario refers to the question of whether, a very large network can be stable for the protocol we consider. Our protocol in this scenario is every arriving link will transmit at full power till completion of file transfer. Hence, it is not apriori clear that qualitative properties of the finite system will carry over to the infinite one. Moreover, we believe that studying the stationary solutions of the infinite regime will help us better understand finer questions in the finite model, such as estimates for tails of delay, which are crucial in dimensioning and operating these networks.

### 1.1.3 Chapter 4 : Technology Diversity - A Framework for Base-Station Association Policies

In Chapter 4, we summarize a research project that culminated in the publication of a paper [8]. This paper considers the problem of how to leverage the presence of multiple cellular operators collaborating to provide service. This work was motivated by recent trends such as Google Fi and the MVNO (Mobile Virtual Network Operators) where a user subscribing to that service 'sees' many different cellular operators operating on orthogonal bands and can potentially connect to any of the serving operators depending on instantaneous payoffs. More concretely, we considered the association problem, wherein a user, depending on the 'information' it has about the instantaneous network chooses a cellular operator and a base-station from that operator to associate with and communicate. We give a precise mathematical formalism for information based on an appropriate filtrations of a probability space. Although the definition is abstract, as noted in the chapter, we can encompass many realistic scenarios into our formalism. Given our framework, we consider the optimization problem of which base-station a user has to associate with by cleverly exploiting available network information. The optimization formulation of the problem is 'parametric', in the sense depends on the statistical distribution of the networks assumed in the model. Hence, it is not totally practical, but nonetheless provides benchmarks on performance evaluation of more realistic schemes. We further propose several heuristic policies that are non-parametric and only 'data-dependent'. Moreover, these heuristic policies require the user to learn only very little information about the instantaneous network state, thus making it easy to implement in practice. Even though our heuristics are sub-optimal, we show that under a certain reasonable asymptotic, our policy coincides with the optimal policy of a mobile phone that knew the entire instantaneous network information. We further see from simulations that the asymptotic needed by our theorem statements can be realized for values of parameters that are practically reasonable. Hence, this heuristic is attractive in practice since it is easy to implement with a provably optimal performance

The introduction of the stochastic optimization framework to design Base-Station association schemes is one of our main contributions in this chapter. To the best of our knowledge, our paper [8] is the first to systematically and rigorously undertake the study of optimal association policies in the stochastic geometry framework for cellular networks within this context. Although our optimal schemes are parametric and difficult to implement, they provide a benchmark for evaluating other practical schemes. We further identify a heuristic that is asymptotically optimal and hence attractive in practice.

### 1.1.4 Chapter 5 - Community Detection on Spatial Random Graphs

In Chapter 5, we shift gears into a different application, namely that of online-social networks. This is both ongoing and proposed future work, wherein we have some partial results and expect to have more complete results by the end of the thesis. Social networks broadly defined are networks between people such as in Facebook, LinkedIn, Twitter etc. The mathematical question we focus on is the problem of 'clustering' or also known as 'Community-Detection' in social networks. The community detection problem is the task of partitioning an underlying population into groups such that all members of a group are similar in some respect and members in different groups are dis-similar. This is a non-trivial problem in practice, since the group membership notion is often not precise, and even when precise, the information about group membership is only known indirectly and needs to be inferred. Hence, a well accepted mathematical framework for community detection is to pose it as an inverse statistical problem. This is the line of thought we follow in this chapter. Although our motivation comes from social networks, we believe our mathematical techniques can be possibly extended to study other network scenarios where similar problems of grouping occurs.

The graph-clustering or the community detection problem is that of partitioning the population given indirect group membership information encoded by a graph whose nodes are the members of the population. This sub-class of the general clustering although seems very specific, is still rich enough to capture many different applications (see [7] and references therein). The most well studied mathematical model of graph-clustering is the Stochastic-Block Model (SBM). This is a random graph model introduced first by [5] in the social sciences literature and has since been re-invented in mathematics, physics and computer-science literature under different contexts. However, it is well known that the SBM has several shortcomings in representing real networks, i.e. it is locally tree-like, whereas real social networks are empirically known to contain several triangles. Indeed, these observations have motivated the development of the 'Latent-Space Social Models' in the social sciences community ( [3], [4]) as a more natural model for social graphs. In this chapter, we consider a natural model combining the SBM and the Latent-Space model to define a new class of planted partition spatial random graphs. We then pose a version of the community detection or the clustering question to our setting and analyze when and how can the graph be clustered. We have partial results giving a necessary and a sufficient condition for community detection. Although both conditions are constructive and algorithmic, these set of conditions are still not tight and in particular is not optimal. In the proposed future work, we plan on extending the analysis to have a better understanding of the fundamental limits in community detection in our setting of spatial random graph. We also plan on setting up experiments to validate some of our modeling choices with real world data.

# Chapter 2

# A Spatial Birth Death Process - Continuum Space Model for spatio-temporal spectrum sharing

The contents of this chapter is from a published paper [6]. The reproduction here are in accordance to IEEE guidelines.

## 2.1 Introduction

In this chapter, we consider a problem of spectrum sharing dynamics in spatio-temporal ad-hoc wireless networks. Ad-hoc wireless networks are infrastructure-less networks in which there is no centralized access point or a base-station. Such networks have received a tremendous amount of attention in the networking and information theory literature, due on the one hand to the increasing ubiquity in modern technology and on the other hand to the mathematical challenges in modeling and performance. The key application that we have in consideration is the Device-to-Device network setting, which is being considered as a potential technology in emerging 5G standard.

The characterization of fundamental limits of spectrum sharing in *static* ad-hoc wireless networks has been considered for long in Network Information Theory under the Interference Channel [10]. By static, we mean that these models do not account for the fact that links may be on or off depending on the wireless traffic. The interference channel model is aimed at capturing the fundamental limit of one shot communication of a packet in an ad-hoc network. Despite the simplicity in the description of the problem, the characterization is notoriously hard and the full characterization is still an open problem in information theory. The progress on this topic can be found in the book by El.Gamal and Kim [11]. In recent years, Stochastic Geometry ( [1], [2]) has emerged as a way of assessing the performance of large-scale wireless networks with links interacting in space through interference. These tools have been very popular to model and analyze wireless system performance for a variety of network architectures including D2D networks, mobile-ad hoc networks [12] and cellular networks [13]. However, the main drawback in these models is that they do not have a notion of temporal interaction and do not allow one to represent random traffic (they usually rely on a full-buffer assumption, i.e., every link always has a packet to transmit).

This additional dimension of interaction among wireless links sharing a common spectrum adds to the complexity of their performance analysis but nonetheless is very crucial to understand network performance. Most prior work aiming at studying the temporal interaction of links model spatial interactions through binary on-off behavior encoded by interference or conflict graphs. The temporal interactions are then modeled using queuing theoretic ideas of flow based models (for ex: [14], [15], [16]). They were initially proposed to study dynamic resource allocation in wired networks ( [17] , [18]), and were subsequently used to model and study wireless networks. The main drawback in employing such temporal models in a wireless scenario is that the spatial and information-theoretic interactions are overly simplified and not captured precisely.

Motivated by this, we propose a new spatial flow model, which uses the continuum space to model link interaction through interference as prescribed by the information-theoretic setting, and also takes into account the interaction of links across time due to traffic variations. Roughly speaking, our model consists of an interacting particle system in space, where links which is a transmitter-receiver pair arrive in space according to a Poisson Point Process in space-time. The transmitter of each link has a file which it wants to transmit to its corresponding receiver. A link exits the network upon completion of this file transfer.

The instantaneous rate at which a transmitter can transmit a file to its receiver is given by the instantaneous Shannon rate, which in turn depends on the geometry of the other transmitters in the network transmitting at that instant to their respective receivers. We study this space-time dynamics to identify a phase-transition in the arrival rate such that each link can be guaranteed to exit in finite time almost surely. The model and the question of phase-transition is formalized in Section 2.2. To the best of our knowledge, the analysis of such continuum space-time models for wireless networks has not been considered so far.

From an information-theoretic viewpoint, one can interpret our model and the phase-transition result as a form of dynamic network capacity. Our network model can be interpreted as consisting of arrivals of a single antenna Gaussian additive noise point-to-point channels in space. At each instant of time, the network is a random realization of an interference network operating under the scheme of treating interference as noise. The point-to-point channels exit the network upon completion of a file transfer i.e. with the departures happening in a space-time correlated way determined by our dynamics which in turn is derived from the capacity region of an interference channel under treating interference as noise. The phase-transition results in Theorems 1 and 2 give the maximum rate of arrival that can be supported in the network under the scheme of treating interference as noise.

Our model also presents a new form of single server queuing network. Based on our model description in Section 2.2, one can come up with two natural queuing model bounds to study the performance of our model. One can construct a worse system by assuming that there is no distance dependent attenuation and all transmitters contribute the same interference to any receiver. This system will predict larger delays than our original system since the interference is higher. Moreover, since there is no geometry, this upper bound system is equivalent to an $M/M/1$ generalized processor sharing system. On the other hand, to come up with lower bounds for delay, one can totally neglect interference and assume that the different links do not interact at all. This assumption will render our model equivalent to an $M/M/\infty$ system. One of our main messages in this chapter is that simplifying our model to any of the above two dynamics which neglects spatial structure to provide bounds on delay leads to estimates for delay which are very poor (as demonstrated in Section V.E ). Thus, we really need to consider the spatial structure as done in Section IV to come up with estimates for delay and performance. The evolution of our model thus presents a novel behavior of stochastic dynamics that cannot be captured by a queuing model that neglects spatial interactions.

## 2.2 Problem Statement

In this section, we describe the mathematical model of the dynamic wireless network which we later analyze. The model is parametrized by $\lambda \in \mathbb{R}_+$, $Q \in \mathbb{R}_+$, $L \in \mathbb{R}_+$ and $l(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ that is assumed to be non-increasing. Roughly speaking, our model of a network is one wherein links which are transmitter-receiver pairs arrive into the network which is Euclidean space. Each transmitter of a link has a file it wants to send to its receiver. The speed or rate at which a transmitter can send its file to the receiver is a function of the positions of other transmitters transmitting files to their respective receivers. Upon completion of file transfer, a link departs from the network. We make the above dynamic description of the network precise.

The wireless links live in $\mathbf{S} \subset \mathbb{R}^2 = [-Q, Q] \times [-Q, Q]$, a square region of the Euclidean plane where $Q$ is a large but fixed *finite* constant. To avoid edge effects, we identify the opposite edges of the square and wrap it around to form a torus. We denote by $|\mathbf{S}|$ as the area of the region $\mathbf{S}$ which is $4Q^2$.

The links arrive into the network as a stationary marked space-time process on $\mathbf{S} \times \mathbb{R}$ with intensity $\lambda$. This marked point-process on $\mathbf{S} \times \mathbb{R}$ is denoted by $\mathcal{A}$. An atom $p \in \mathbb{Z}$ of $\mathcal{A}$ represents the receiver and is denoted by $(x_p, b_p)$. $x_p \in \mathbf{S}$ denotes the spatial location of receiver $p$ and $b_p \in \mathbb{R}$ denotes the time of arrival into the network of receiver $p$. Hence, one can represent the point process $\mathcal{A}$ as $\mathcal{A} = \sum_{p \in \mathbb{Z}} \delta_{(x_p, b_p)}$, where $\delta_{(x,b)}$ refers to the Dirac-mass at $(x, b) \in \mathbf{S} \times \mathbb{R}$. To each point $p$ of $\mathcal{A}$, we associate a vector mark of $(y_p, L_p)$, where $y_p \in \mathbf{S}$ and $L_p \in \mathbb{R}^+$, where $y_p$ refers to the location of the transmitter of receiver $p$ and $L_p$ denotes the file-size which the transmitter of $p$ wants to send to the receiver of $p$. We refer to the pair $(x_p; y_p)$ as link $p$ whose receiver is in location $x_p$ and transmitter in location $y_p$. The length of link $p$ is denoted by $T_p := ||x_p - y_p||$.

The set of links present or *alive* in the network at time $t$ is denoted by $\phi_t$ i.e. $\phi_t = \{(x_1; y_1), ..., (x_{N_t}; y_{N_t})\}$, where $N_t$ is the number of links alive in the network at time $t$. The exact dynamics describing which links are present at a particular time $t$ will be specified in the sequel. More formally, $\phi_t = \sum_{i=1}^{N_t} \delta_{x_t}$ is a point-process on $\mathbf{S}$ of receivers marked with the location of their transmitters. We use the terminology "*configuration of links*" to refer to a marked point-process on $\mathbf{S}$ (atoms representing the receiver locations) with its marks (representing its corresponding transmitter locations) in $\mathbf{S}$. We denote by $\phi_t^{Tx} = \{y_1, \cdots, y_{N_t}\}$, the point-process of transmitters present at time $t$ in the network and by $\phi_t^{Rx} = \{x_1, \cdots, x_{N_t}\}$, the point process of receivers at time $t$ in the network.

The transmitter of each link $p$ has a file of size $L_p$ measured in bits which needs to be communicated to its receiver. The transmitter sends this file to its receiver at a time varying rate given by the instantaneous Shannon rate. We thus, define the rate of file transmission by a transmitter to its receiver as

$$R(x, \phi) = C \log_2 \left( 1 + \frac{l(||x - y||)}{\mathcal{N}_0 + \sum_{u \in \phi^{Tx} \setminus \{y\}} l(||x - u||)} \right), \tag{2.1}$$

where $l(\cdot)$ is the path-loss function. $C$ is a positive constant and $\mathcal{N}_0$ is a positive constant denoting the thermal noise. We denote by 'interference' seen at $x$ due to configuration $\phi$ as to mean

$$I(x, \phi) = \sum_{u \in \phi^{Tx} \setminus \{y\}} l(||x - u||). \tag{2.2}$$

This setup now allows one to precisely define the network dynamics. A link arriving with receiver in location $x_p \in \mathbf{S}$ and its transmitter at location $y_p \in \mathbf{S}$ at time $t_p$ with file of size $L_p$ leaves the network at time $d_p$ given by the following recursive definition

$$d_p = \inf \left\{ t > b_p : \int_{u=t_p}^{t} R(x_p, \phi_u) du \geq L_p \right\}. \tag{2.3}$$

In the above equation, $\phi_u$ denotes the point process of all links "alive" at time $u$ i.e. $\phi_u^R = \sum_{p \in \mathbb{Z}} \delta_{x_p} \mathbf{1}_{\{u \in [b_p, d_p]\}}$ and $\phi_u^T = \sum_{p \in \mathbb{Z}} \delta_{y_p} \mathbf{1}_{\{u \in [b_p, d_p]\}}$ where $\delta_x$ denotes to the Dirac-measure at location $x \in \mathbf{S}$. We refer to the time instant $b_p$ as the "birth" time of link $p$ and $d_p$ as the "death" time of link $p$. This is the justification for calling this dynamics a "spatial birth-death" model, i.e. this transmitter-receiver pair is "born" at time $b_p$ and "dies" at time $d_p$ and leaves the network.

### 2.2.1 Mathematical Assumptions

All the analysis and results rely on the following assumptions on the system model presented in the previous section.

1. The link arrival process is a time-space stationary Poisson Point Process of intensity $\lambda$. The probability of an arrival of a receiver in an infinitesimal location $dx$ in an infinitesimal time interval $dt$ is $\lambda dx dt$.

2. The file sizes of each transmitter are i.i.d. and exponentially distributed with mean $L$ bits.

3. The transmitter location $y$ of a receiver at $x$ is assumed to be distributed uniformly and independently of everything else on the perimeter of a ball of radius $T$ centered at $x$. In particular, the received signal power at any receiver is $l(T)$.

4. The thermal noise power $\mathcal{N}_0 > 0$ is a fixed constant.

5. The path-loss function is bounded and non-increasing with $l(0) = 1$. This is a reasonable assumption since energy is only dissipated on traveling through space and the received energy can be no larger than the transmit energy.

These assumptions (especially the statistical ones) are imposed primarily for mathematical tractability. It is well known, at least in the context of the Internet, that file sizes are Pareto [19] and it would make modeling sense to assume heavy-tailed file sizes. We relax the statistical assumption on exponential file-sizes in the simulation studies in the full paper [6]. Nonetheless, studying the system under the Markovian statistical assumptions form a necessary first step before considering the general case.

In our model, we have that all links have the same length of $T$. This is commonly referred to as the 'Dipole-Model' of an ad-hoc wireless network [20]. An interesting limiting case is that of $T = 0$. This corresponds to the physical case of when the link lengths are very small compared to the size of the network. In this limiting case, the point process $\phi_t$ is simple and unmarked since the transmitter and receiver locations are identical, and the signal power is $l(0) = 1$. The interference function at a point $x$ from configuration $\phi$ is then $I(x, \phi) = \sum_{y \in \phi \setminus \{x\}} l(||y - x||)$. We mention this limiting case here as it will help us to get a much better understanding of what our theoretical results imply, especially that of clustering (defined later in Definition 1). However, all of our mathematical results are valid for general arbitrary link distances $T$.

Although the assumptions may render the model somewhat specific, it still presents a formidable mathematical challenge and captures the key features of a spatio-temporal dynamic wireless network. Most prior works incorporating spatial interference circumvent this mathematical difficulty by making 'full-buffer' assumptions which is equivalent to assuming no temporal interactions. Our results, especially the closed form expressions for approximating of delay are the first in the context of spatio-temporal wireless network models to the best of our knowledge.

The statistical assumptions, namely the Poisson arrival process and i.i.d. exponential file sizes imply that the process $\phi_t$ is a continuous time measure-valued Markov Chain on the state space of marked simple counting measure on $\mathbf{S}$ denoted as $\mathbf{M}(\mathbf{S})$ [?]. More precisely, the process $\phi_t$ is a piece-wise constant jump Markov Process i.e., from a time $t$, the next *change* in the configuration will occur after an exponentially distributed time duration with rate $\lambda|S| + \frac{1}{L}\sum_{x\in\phi_t} R(x,\phi_t)$. This interpretation follows since births occur at the epochs of an exponential clock with rate $\lambda|S|$ and the death rate of any receiver $x$ in configuration $\phi$ is $\frac{1}{L}R(x,\phi)$ which is independent of everything else. The assumption $Q < \infty$ ensures that $\phi_t$ is a piece-wise constant jump process. Extending the analysis of stability to the case of $\mathbf{S} = \mathbb{R}^2$ is way more challenging and is left for future work. The large torus is meant to emulate the Euclidean space. The fact that it is similar to the Euclidean space (in terms of interference field and hence birth and death dynamics) justifies our use of the Palm calculus of the Euclidean space rather than that of the torus in some derivations.

The first natural question we ask about $\phi_t$ is that of time ergodicity which we address in the next section. Time ergodicity implies that the process $\phi_t$ admits an unique steady-state in which the links form a stationary and space-ergodic point process on $\mathbf{S}$. Moreover, since $\mathbf{S}$ is a compact set, the stationary-regime when it exists will put only finitely many points in $\mathbf{S}$ at any given instant almost-surely. Denote by $\phi_0$ the steady-state point-process of links i.e. the links that are "alive" or active in steady-state. $\phi_0$ is a point-process on $\mathbf{S}$ with atoms representing the locations of receivers and marks representing the relative transmitter locations.

Denote by $\beta$ the density of links present in the network in steady-state (assuming it exists). More formally, $\beta$ denotes the intensity of the receiver point-process $\phi_0^{Rx}$ (which is the ground point process of $\phi_0$) on $\mathbf{S}$ when the dynamics is in steady state. Note that the intensity of the transmitter point-process $\phi_0^{Tx}$ in steady-state is also $\beta$ since every receiver in the model has exactly one transmitter. The distribution of the relative location of the transmitter of a typical receiver of $\phi_0^{Rx}$ is uniform on the perimeter of a ball of radius $T$ around this receiver. However, the transmitter locations across different receivers of $\phi_0^{Rx}$ are not independent due to the correlation (clustering) induced by the dynamics.

The interpretation of time ergodicity is also connected to the phase-transition of mean delay. Little's law for this dynamics yields $\beta = \lambda W$, where $W$ is the average sojourn time of a typical link i.e., $W = \mathbb{E}[d_0 - b_0]$; which follows from PASTA [21]. The process $\phi_t$ being time ergodic in our model is equivalent to asserting that $W < \infty$, i.e. finite mean delay for a typical link in the network. This interpretation is what we allude to in the system insight section which allows one to evaluate how frequently in space and time should the traffic arrival process be (i.e. how large $\lambda$) can be for the network to provide finite mean-delay to all links.

## 2.3 Main Theoretical Results

The main theoretical results of our paper are on the time-ergodicity (or stability) conditions of the dynamics $\phi_t$ and on a certain structural characterization of the steady-state point process of $\phi_t$ whenever it exists. The proofs of the theorems are presented in the Appendix.

### 2.3.1 Stability Criterion

We state our main theoretical results on the stability criterion (i.e. time ergodicity) of the dynamics.

**Theorem 1.** *If* $\lambda > \frac{Cl(T)}{\ln(2)La}$*, then the Markov Chain* $\phi_t$ *admits no stationary regime.*

We see from the proof (in Appendix $A$ of [6]) that this theorem only needs the weaker assumption that $l(\cdot)$ be such that $l(r) < \infty$ for all $r > 0$. This indeed is a weaker assumption than assuming that the function $l(\cdot)$ is bounded. Thus, we have as immediate corollary to this theorem:

**Corollary 1.** *For the path-loss model $l(r) = r^{-\alpha}$, $\alpha \geq 2$, for all $\lambda > 0$, and all mean file sizes, the process $\phi_t$ admits no stationary-regime.*

*Proof.* This follows since the integral $\int_{x \in \mathbf{S}} l(||x||)dx$ diverges for the function $l(r) = r^{-\alpha}$ for all $\alpha \geq 2$. □

The next result provides a tight condition for time ergodicity.

**Theorem 2.** *If $\lambda < \frac{Cl(T)}{\ln(2)La}$, then the Markov Chain $\phi_t$ is time ergodic, i.e. has an unique stationary regime.*

The two theorems identify the exact critical arrival rate $\lambda$ for ergodicity as $\lambda_c = \frac{Cl(T)}{L\ln(2)a}$. We however refrain from studying the critical case as it is technically more subtle. In the sequel, whenever we refer to $\phi_0$, we implicitly assume $\phi_t$ is ergodic, i.e. the condition $\lambda < \frac{Cl(T)}{\ln(2)La}$ holds.

### 2.3.2 Clustering

In this section, we state the main structural characterization of the steady-state point process $\phi_0$ when it exists i.e. when $\lambda < \frac{Cl(T)}{L\ln(2)a}$. We need the following definition of clustering.

**Definition 1.** *(CLUSTERING) Let $\phi$ be a stationary configuration of links, i.e. it is a stationary marked point-process on $\mathbf{S}$ with its marks in $\mathbf{S}$. Then $\phi$ is said to be clustered if for all bounded, positive, non-increasing functions $f(\cdot) : \mathbb{R}^+ \to \mathbb{R}^+$, the following inequality holds*

$$\mathbb{E}^0_{\phi}[F(0,\phi)] \geq \mathbb{E}[F(0,\phi)], \tag{2.4}$$

*where $F$ is the shot-noise defined as follows. For any atom (receiver) $x \in \phi$ with its corresponding mark (transmitter) $y \in \mathbf{S}$, the shot noise $F(x, \phi) := \sum_{T \in \phi^{Tx} \setminus \{y\}} f(||T - x||)$.*

**Theorem 3.** *If the dynamics $\phi_t$ is ergodic, then the steady state point process $\phi_0$ is clustered.*

By substituting $f(\cdot) = l(\cdot)$ in Equation (2.4) , we get that the mean of the interference measured at any uniformly randomly chosen receiver in the steady-state point process (this is the interpretation of the Palm probability) is larger than the mean of the interference measured at any uniformly randomly chosen location of space in $\mathbf{S}$.

To understand why the above definition is a form of clustering, consider the case $T = 0$ which gives a clearer picture. In this case, Theorem (3) gives a clustering comparison of $\phi_0$ with a Poisson Point Process (PPP) of same intensity. Let $\psi$ be a PPP of the same intensity as $\phi_0$. Then, from Slivnyak's theorem (Theorem 1.4.5, [20]), one can rewrite the inequality in (2.4) as

$$\mathbb{E}^0_{\phi_0}[F(0, \phi_0)] \geq \mathbb{E}^0_{\psi}[F(0, \phi_0)], \tag{2.5}$$

where $\mathbb{E}^0_{\psi}[F(0, \phi_0)] = \mathbb{E}[F(0, \psi)]$ follows from Slivnyak's theorem which is equal to $\beta \int_{x \in \mathbf{S}} f(||x||)dx$ from Campbell's Theorem (Theorem 1.4.3, [20]). Slivnyak's theorem essentially gives that the PPP has no clustering i.e. the Inequality 2.4 is an equality. Hence, we automatically have a shot noise comparison of the steady state point process $\phi_0$ with a PPP.

The comparison with a PPP also gives us a comparison of the Ripley K-function [22] of $\phi_0$ with that of a PPP. The Ripley K-function $K_\phi(\cdot) : \mathbb{R}^+ \to \mathbb{R}^+$ of a point-process $\phi$ is defined as $K_\phi(r) = \frac{1}{\beta}\mathbb{E}^0_\phi[\phi(B(0, r)) - 1]$ where $\beta$ is the intensity of $\phi$ and $\mathbb{E}^0_\phi$ is the Palm probability measure of $\phi$. This function can be interpretted as the mean number of points (scaled by the intensity of the point-process) within distance $r$ to the origin *conditioned* on a point of $\phi$ to be present at the origin. The Ripley K-function is commonly used in statistical analysis of point-patterns to identify if an empirical data-set exhibits statistical clustering [22]. Based on the shot-noise comparison with a PPP, we have the following corollary.

**Corollary 2.** *Assume $\phi_t$ is in steady-state and $T = 0$. Denote by $\beta$ to be the intensity of $\phi_0$ and $\psi$ to be a PPP on $\mathbf{S}$ with intensity $\beta$. Then, $K_{\phi_0}(r) \geq K_\psi(r)$.*

*Proof.* Consider $f(x) = \mathbf{1}(x \leq r)$ in Theorem 3. □

We will use Ripley K-function in the simulations to compare the point process $\phi_0$ with a PPP to derive a bound on the intensity $\beta$ of $\phi_0$ as a function of $\lambda$, $L$ and $l(\cdot)$.

Intuitively, it is not surprising to expect a clustered point-process in steady state. An arriving link gets lower rate if it is in a crowded area of transmitters, due to interference. This arriving link also causes more interference to the cluster of links already present thereby causing more interference and slowing everyone down. This reinforcement of service slow-down is actually the fundamental reason making the system always unstable in the power law attenuation function case. More generally, when $\phi_t$ is sampled in steady-state, it is expected to be clustered as formalized by Theorem 3. A snapshot of the point-process $\phi_0$ is presented in Figure 2.1 which gives a visual illustration of the clustering.

### 2.3.3 A Word on the Proofs

Our main contribution is in developing a technique based on rate-conservation principle to be able to prove these results on dynamic point-processes. To prove Theorem 1, we write equations conserving several natural quantities and then finding an algebraic contradiction. Thus, if $\lambda > \frac{Cl(T)}{\ln(2)La}$, then the rate-conservation equations do not hold. This implies that the Markov process $\phi_t$ has no stationary regime. As a consequence, we get through Harris's inequality, the proof of Theorem 3. f

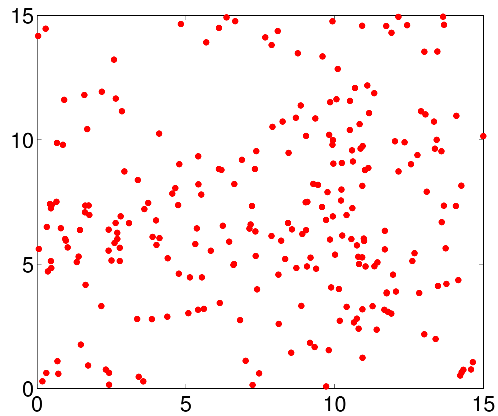


Figure 2.1: A sample of $\phi_0$ when $\lambda = 0.99$ and $l(r) = (r+1)^{-4}$. This is a visual representation of the clustering of points.

The proof of Theorem 2 is a bit involved. We first prove the result for $T = 0$, i.e. by assuming link length is 0, before considering the general link length. For the case of $T = 0$, we reduce the problem from continuum space to discrete space by an appropriate monotone coupling argument. This coupling essentially follows from the monotonicity in the dynamics, which gives that rate give in Equation (2.1) is non-decreasing in the interference. Then, we analyze the discrete space model as a multidimensional generalized processor sharing network using the Fluid Limit approach of [23] and [24]. The details can be found in Appendix $B$ of [6]. We then provide a simple argument to extend the analysis for $T = 0$.

## 2.4 Performance Analysis - Steady State Formulas

In this section, we propose two heuristic formulas for $\beta$ the intensity of the point process $\phi_0$ as a function of $\lambda$. Note that a heuristic formula for $\beta$ gives a heuristic formula for mean delay $W$ through Little's Law ($\beta = \lambda W$).

We propose two formulas - $\beta_f$ called the *Poisson Heuristic* and $\beta_s$ called *Second-Order heuristic* to approximate $\beta$ the intensity of the steady-state point process $\phi_0$. We show that subject to a natural conjecture (Conjecture 1), $\beta_f$ is a lower bound on $\beta$. We see from simulations however that $\beta_s$ is a much better approximation of $\beta$ compared to $\beta_f$. Both formulas are derived based on approximately evaluating the following Equation which we establish in Equation (18) in Appendix $A$ of the full paper [6].

$$\lambda L = \beta \mathbb{E}^0_{\phi_0}\left[\log_2\left(1 + \frac{l(T)}{\mathcal{N}_0 + I(0,\phi_0)}\right)\right]. \tag{2.6}$$

### The Poisson Heuristic

The Poisson heuristic formula $\beta_f$ is given by the largest solution to the following fixed point equation

$$\lambda L = \frac{\beta_f}{\ln(2)}\int_{z=0}^{\infty}\frac{e^{-\mathcal{N}_0 z}(1 - e^{-zl(T)})}{z}e^{-\beta_f q(z)}dz, \tag{2.7}$$

where $q(z) = \int_{x\in\mathbf{S}}(1 - e^{-zl(||x||)})dx$. This formula is obtained by approximating the expectation in Equation (2.6) by assuming the following "*Independent Poisson heuristic*". We assume that $\phi_0$ is an independently marked Poisson-Point process with the transmitter locations of different receivers in $\phi_0$ being independent. Since the transmitter locations are assumed to be independent, the process $\phi_0^{Tx}$ will also be a PPP in this Poisson heuristic. We state the following lemma without proof from [25] which is useful in computing the expectation under the Poisson assumption.

**Lemma 1.** *Let $X, Y$ be non-negative and independent Random Variables. Then,*

$$\mathbb{E}\left[\ln\left(1 + \frac{X}{Y+a}\right)\right] = \int_{z=0}^{\infty}\frac{e^{-az}}{z}(1 - \mathbb{E}[e^{-zX}])\mathbb{E}[e^{-zY}]dz.$$

We can then explicitly compute the expectation in Equation (2.6) by letting $X = l(T)$ to be deterministic and $Y = I(0,\phi_0)$ as follows

$$\lambda L = \beta_f \mathbb{E}^0_{\psi}\left[\log_2\left(1 + \frac{l(T)}{\mathcal{N}_0 + I(0)}\right)\right]$$

$$\stackrel{(a)}{=} \beta_f \mathbb{E}_\psi \left[ \log_2 \left( 1 + \frac{l(T)}{\mathcal{N}_0 + I(0)} \right) \right]$$
$$\stackrel{(b)}{=} \frac{\beta_f}{\ln(2)} \int_{z=0}^{\infty} \frac{e^{-\mathcal{N}_0 z}(1 - e^{-zl(T)})}{z} e^{-\beta_f q(z)} dz,$$

where $q(z) = \int_{x \in \mathbf{S}} (1 - e^{-zl(||x||)})dx$ and $\psi$ is a Poisson Point Process on $\mathbf{S}$ with intensity $\beta_f$. The equality $(a)$ follows from Slivnyak's theorem and the equality $(b)$ follows from Lemma 1 and the formula for the Laplace functional of a Poisson Point Process. The subscript $f$ refers to the computation of the density under this Poisson heuristic. This establishes the formula in Equation (2.7).

We now make the following conjecture on the higher-order moment measures of $\phi_0$, which we will leverage to show that $\beta_f$ is a lower bound on $\beta$.

**Conjecture 1.** *Let $\phi_0$ be the point process on $\mathbf{S}$ corresponding to the stationary distribution of $\phi_t$ with intensity $\beta$. Denote by $\psi$ to be an independently marked Poisson Point Process on $\mathbf{S}$ with intensity $\beta$. The mark of any atom $x$ of $\psi$ is a point $y$ drawn uniformly on the perimeter of a circle of radius $T$ around $x$. Then, for any $s > 0$, we have $\mathbb{E}^0_{\phi_0}[e^{-sI(0;\phi_0)}] \leq \mathbb{E}^0_{\psi}[e^{-sI(0;\psi)}]$.*

Note that from Slivnyak's theorem we also have $\mathbb{E}^0_{\psi}[e^{-sI(0;\psi)}] = \mathbb{E}_{\psi}[e^{-sI(0;\psi)}]$. This conjecture which is validated through simulations in Figure 2.2, is a slightly different statement on the structural characterization of $\phi_0$ than stated in Theorem 3. This conjecture gives that the Laplace transform of the interference measured at a typical receiver of $\phi_0$ is larger than that measured at a typical receiver of an equivalent PPP. In general, whenever we have ordering of the mean, then we have ordering of the Laplace Transform only as $s \to 0$. This ordering for the Laplace transform as $s \to 0$ follows from Taylor's expansion that $e^{-sx} \approx 1 - sx$ as $s \to 0$. However, in our case, we believe that the ordering on the Laplace transform holds for all $s \geq 0$ but we are unable to prove so. The intuition for this follows from the pictorial interpretation that there are roughly the same number of interfering transmitters around a typical receiver in $\phi_0$ and $\psi$ since the intensities of $\phi_0$ and $\psi$ are the

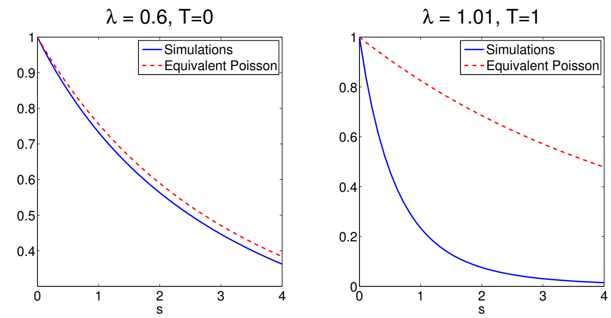


Figure 2.2: A plot comparing the functions $\mathbb{E}^0_{\phi_0}[e^{-sI(0;\phi_0)}]$ and $\mathbb{E}^0_{\psi}[e^{-sI(0;\psi)}]$, for $l(r) = (r+1)^{-4}$.

same. However, the interfering transmitters are closer to the typical receiver in $\phi_0$ as compared to in $\psi$. This intuition follows from Corollary 2 where we had ordering of the Ripley-K function of $\phi_0$ and $\psi$. This pictorial interpretation then gives an intuition for the conjecture since, the interference is the sum of attenuated powers from interfering transmitters where the attenuation is through a function that is non-increasing with distance. Thus, $I(0)$ is the sum of roughly the same number of terms in both $\phi_0$ and in $\psi$, but each of the terms are slightly larger in $\phi_0$ than in $\psi$. This interpretation can possibly be made rigorous in the asymptotic regime as $\lambda \uparrow \lambda_c$ by alluding to certain concentration phenomenon. However, we see from simulations that this conjecture holds true for all regimes of $\lambda$. This conjecture is further substantiated in Figure (2.3) which underpins Proposition 4.

The ordering of the mean does not always imply the ordering of Laplace transforms in general. As a very simple example consider two random variables $X$ and $Y$ where $X$ takes values $\{1, 2, 3, 4\}$ with probabilities $\left\{\frac{1}{6}, \frac{1}{3}, \frac{1}{6}, \frac{1}{3}\right\}$ and $Y$ is deterministic and takes value of 2. Here $\mathbb{E}[X] = \frac{8}{3}$ and $\mathbb{E}[Y] = 2$. However, for $s = 1.1$, $\mathbb{E}[e^{-sX}] > \mathbb{E}[e^{-sY}]$. More generally if $\mathbb{E}[X] \geq \mathbb{E}[Y]$ but the higher order moments are ordered in the opposite direction, then one cannot expect an ordering on the Laplace-transform.

**Proposition 4.** *Subject to Conjecture (1), we have that* $\beta \geq \beta_f$, *where* $\beta_f$ *is the largest solution of Equation (2.7).*

*Proof.* Let $g(\beta) = \beta\mathbb{E}^0_{\phi_0}[R(0;\phi_0)]$ (where $\phi_0$ has intensity $\beta$) and let $p(\beta) = \beta\mathbb{E}^0_{\psi}[R(0;\psi)]$

where $\psi$ is a PPP on $\mathbf{S}$ with intensity $\beta$. Rate-conservation equation (2.6) gives that $\lambda L = g(\beta)$ and our heuristic computation is $\lambda L = p(\beta_f)$. From our conjecture and Lemma 1, we have the inequality $g(\beta) \leq p(\beta)$. The function $g(\beta) = \beta \mathbb{E}^0_{\phi_0}[R(0; \phi_0)]$ is monotone non-decreasing in $\beta$ as it describes the true dynamics through the equation $\lambda L = g(\beta)$. The monotonicity of $g(\cdot)$ along with the inequality $g(\beta) \leq p(\beta)$ gives the performance bound $\beta \geq \beta_f$. □

Proposition 4 gives that $\beta_f|\mathbf{S}|$ is a lower bound on the mean number of links present in the network in steady state and $\frac{\beta_f}{\lambda}$, as a lower bound on mean-delay of a typical link.

The Poisson heuristic completely ignores the spatial clustering we established in Theorem 3 and assumes complete-spatial randomness. Since it does not account for the clustering it underestimates the typical interference seen at a receiver and therefore predicts a lower density of links. We see through simulations, that this heuristic is poor (i.e. the gap between $\beta$ and $\beta_f$ is large) in certain traffic regimes (Figure 2.3). This is not surprising as one cannot neglect the effect of spatial correlations except in asymptotic regimes of heavy and light-traffic (detailed later). Motivated by the poor performance of the Poisson heuristic in certain regimes, we propose a "second-order heuristic" $\beta_s$ which takes into account the spatial correlations by considering an approximation of the second-order moment measure of $\phi_0$. We see through simulations (Figure 2.3) that $\beta_s$ is a much better approximation of $\beta$ than $\beta_f$ in all traffic regimes.

### Second-Order Heuristic

We propose a heuristic formula $\beta_s$ for approximating $\beta$ in Equation (2.8). For all values of $T$, $\beta_s$ is given by

$$\beta_s = \frac{\lambda L}{C \log_2 \left( 1 + \frac{l(T)}{\mathcal{N}_0 + I_s} \right)}, \tag{2.8}$$

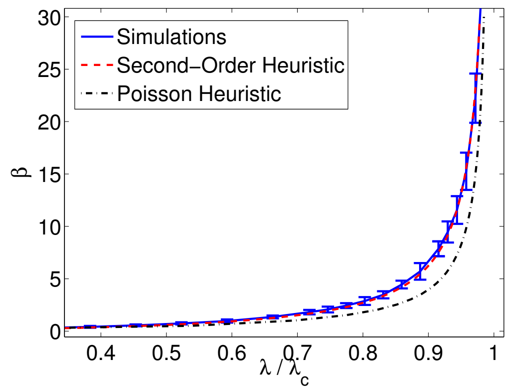


Figure 2.3: The performance plot with 95% confidence interval when $T = 0$ and $l(r) = (r + 1)^{-4}$.

where $I_s$ is the smallest solution of the fixed-point equation

$$I_s = \lambda L \int_{x \in \mathbf{S}} \frac{l(||x||)}{C \log_2 \left(1 + \frac{l(T)}{\mathcal{N}_0 + I_s + l(||x||)}\right)} dx. \tag{2.9}$$

We call the heuristic in Equation (2.8) a *second-order heuristic* since it follows from an approximation of the second-order moment measure of $\phi_0$ as follows. Let $I_s$ denote the interference of a typical point at $\phi_0$ and *assume* it is non-random and equal to its mean. Then, Equation (2.8) follows from Rate-Conservation in Equation (2.6). To compute $I_s$, we use the following approximation of the second order moment measure $\rho^{(2)}(x, y)$ of $\phi_0$ as

$$\rho^{(2)}(x, y) \approx \frac{\beta \lambda L}{C \log_2 \left(1 + \frac{l(T)}{\mathcal{N}_0 + I_s + l(||x-y||)}\right)}. \tag{2.10}$$

Intuitively, the approximation is a form of cavity approximation which can be understood as follows. Two points at locations $x$ and $y$ will each "see" an interference of $I_s$ which is the interference of a typical point plus the additional interference caused by the presence of the other point. Using the above interpretation of interference, Equation (2.10) is a form of Rate-Conservation on the pair of points at $x$ and $y$. The average increase of the pair happens at rate $2\lambda\beta$ and the average decrease of the pair happens at the rate equal to the sum of rates (since file-sizes are i.i.d. exponential) received by points $x$ and $y$ which is approximately $2(C/L)\log_2(1+\frac{l(T)}{\mathcal{N}_0+I_s+l(||x-y||)})$ from the cavity approximation. Now, using the fact that $\mathbb{E}^0_{\phi_0}[I_0] := I_s = \frac{1}{\beta}\int_{x\in\mathbf{S}} l(||x||)\rho^{(2)}(x,0)dx$, we get Equation (2.9) from Equation (2.10).

The heuristic $\beta_s$ is compared against the true $\beta$ and the Poisson heuristic $\beta_f$ in Figure 2.3. The second-order heuristic performs much better compared to the Poisson-heuristic as it takes into account some notion of spatial correlations which the Poisson heuristic completely ignores.

## 2.5 Discussion and Conclusion

In this chapter, we presented a simple mathematical model of spatial birth-death process that modeled spectrum sharing. To the best of our knowledge, this is the first work to consider an interacting space-time model to study ad-hoc wireless networks. From a mathematical contribution, the model presented here is an example of a spatial dynamics that does not behave like an equivalent queue. The reason is no equivalent queue representation can capture the clustering phenomenon we proved in Theorem 3 and is evidenced more through simulations in the main paper.

This chapter is just a first step in understanding spatio-temporal wireless interactions. In particular, having a more refined understanding of the clustering phenomenon through the moment-measures of the steady state point process will be crucial in designing medium access protocols. Another interesting question in this model is that of tails of delays experienced by a typical point in steady-state. Such estimates are useful for network provisioning and dimensioning. In the next chapter, we outline a certain extension of this model that will be another small step in understanding more about such spatial birth-death dynamics.

# Chapter 3

# Interacting Queues on $\mathbb{Z}^d$ - An infinite plane model for Spatio-Temporal spectrum sharing

This is proposed future work. In this chapter, we detail the model and the motivations for studying this topic and list some partial progress. We will also list some questions we hope to answer in the final thesis.

## 3.1 Introduction

In the previous chapter, we proposed and studied a continuum space birth death process with the points living in some *compact* set $\mathbf{S}$ of $\mathbb{R}^d$. However, we would first want to understand the behavior of the spatial birth-death dynamics described in the previous section on the infinite plane $\mathbb{R}^2$. Understanding the time-stationary regime of our spatial birth-death dynamics on the whole plane will probably give us a better handle on understanding questions of clustering and delay tails we alluded to in the previous chapter. In section 3.2, we will describe the precise model we study and the relevant questions we want to ask and answer as part of the dissertation.

## 3.2 Problem Statement

In this project, we consider a discrete space model, but on an infinite state-space. In view of Lemma (2) we proved in [6], a discrete spatial location can be assumed without loss of generality to study stochastic stability. The model is parametrized by $d \in \mathbb{N}$, $\lambda \in \mathbb{R}_+$, $\{a_i\}_{i\in\mathbb{Z}^d}$ where for all $i \in \mathbb{Z}^d$, $a_i \geq 0$ and all but finitely many $a_i$'s are non-zero and $a_i = a_{-i}$.

We have an infinite collection of queues indexed by $\mathbb{Z}^d$. Each queue is fed by independent arrivals of a Poisson Point Process of intensity $\lambda$. At any time $t \geq 0$ and any $i \in \mathbb{Z}^d$, $x_i(t)$ denotes the number of customers in queue $i$ at time $t$. At any time $t$, the rate of departure from queue $i$ is $\frac{x_i(t)}{\sum_{j\in\mathbb{Z}^d} a_j x_{i+j}(t)}$, provided $x_i(t) > 0$, and 0 otherwise. Furthermore, the departure process is independent across queues. This is why we call our model as an 'infinite interacting' queues problem, where the interaction is in the departure process through the departure rates that depend on each other. This interaction depends on the graph topology in which the queues lie (which is the grid $\mathbb{Z}^d$ in our case) and the instantaneous queue lengths.

The above model is a natural representation of the continuum space dynamics from the previous chapter to the whole plane. The fact that space is discretized is not so serious thanks to the monotonicity argument we proved in Lemma 2 in [6]. Indeed, we moved to an equivalent discrete space model to provide a sufficient condition in the continuum space dynamics of the previous chapter. The other simplification is that we replaced the logarithm of 1 plus the SINR with just SIR. In other words, we assume in the current chapter that the death rate of a particle is proportional to its SIR rather than that given by Shannon's formula. We do this approximation as a first step in understanding the dynamics before considering a more sophisticated formula for the dynamics. From a physical layer perspective, our rate function can be considered as the 'low-SINR' regime of operation of the wireless network.

## 3.3 Practical Motivations for an Infinite Model

An immediate concern with our model is the fact that the network is represented through an infinite number of queues. This seems absurd at first since the network, no matter how large is still finite. Our motivation of considering an infinite network is to effectively *model* a *large* wireless system, as the model provides a good framework for abstraction and design. An infinite model usually exhibits nice phase-transitions which can provide qualitative inputs to design, and are often computationally and analytically tractable due to the symmetries. In our particular case, it is possible that for small values of $\lambda$, for all reasonable initial conditions, the dynamics converges weakly to a stationary regime, and after a threshold $\lambda$, only certain mild initial conditions could possibly converge to the stationary distribution, and of-course above the stability threshold, no initial conditions will converge to the stationary distribution. The intermediate regime, where certain initial conditions converge to the stationary regime while other initial conditions do not is qualitatively interesting as it indicates the presence of certain long-range spatial and temporal correlations in the network. Note that in any finite model, from classical results in Markov Processes, such a regime will not be present. In our previous model, we saw the dichotomy that either all initial conditions converge to the unique stationary distribution, or no initial conditions will converge weakly. We also gave an exact phase-transition for this dichotomy. However, in the infinite model, there is a possibility of a regime in which certain initial conditions converge while others do not. From a practical perspective, operating a network in such a regime is unfavorable. This is because if a certain bad-event occurs (say of heavy clustering), then this could have lasting ramifications, i.e. the network cannot cope with this rare occurrences of congestion. In concise terms, the infinite model is the right abstraction to consider *scalability* of the network and the protocol. A network or a protocol is said to be scalable if it can successfully work as the size of the network grows large. Understanding whether a design is scalable is of paramount importance in large-scale deployments of networks, and the infinite network model is then the right mathematical abstraction to consider this question. From a mathematical perspective, the infinite model can shed more light on the clustering and the rare bad-events in the finite model, both of which are crucial to predict and mitigate in a real deployments. The potential for such phase-transitions and mathematical phenomena makes the infinite model attractive to study both from a mathematical and practical point of view.

### 3.3.1 Related Work

We are not the first to consider such an infinite dimensional interacting queue model for representing large engineering systems. Analyzing large interacting queuing systems are gaining popularity in modeling modern large scale infrastructure such as content-delivery (for eg. [26]), cloud-storage (for eg, [27]), computing clusters (for eg. [28]) and in wireless systems (for eg. [29]) to name a few. Typically, these models, feature $n$ queues, where $n$ is a scaling parameter with $n \to \infty$ as the right interesting asymptotic to study. The classical queuing models are of the 'power-of-d' choice paradigm or also known as the supermarket model and the slotted aloha type model. The supermarket model has received a lot of attention recently due to its applicability in the design of load balancing algorithms in cloud and data-center networks. The supermarket model was introduced by [30] with a lot of recent work from Bramson et.al [31] and Srikant et.al [32]. In this framework, the interaction between queues are in the arrivals, i.e. an arrival is placed into a queue depending on the queue's instant state. The slotted aloha model of [29], the arrivals are non-interactive, but the interaction happens in the departures. In the simplest variant, each non-empty queue attempts a transmission in each slot independently with some probability. A packet successfully exits a queue, if that queue is the only one to attempt to transmit in that slot. In both of these models, studying the system for any fixed $n$ is hard, due to the correlations across queues. However, the limiting system as $n \to \infty$ admits tractable analysis owing to a 'propagation of chaos' type phenomena and also is more applicable to reality where the systems are large. Indeed, the major focus in analyzing those models is to establish in some sense the propagation of chaos which enables one to be able to switch the 'limits' in time and space. The limits in our model also commute, but without exhibiting asymptotic independence. Infact, one of our major achievements is to analyze these interacting queuing systems without the need for independence assumption across queues. One of our main results (Theorem 3 adapted to the infinite setting) is that the stationary distribution exhibits a form of clustering.

Our model, also broadly fits into the theme of studying 'large interacting queuing networks' coming from an application in wireless networks. However, the wireless ad-hoc network application is fundamentally different from that mentioned in the previous chapters. For one. in the current wireless scenario, the arrivals are independent and the interaction happens in the departure process. But, a more structural difference is that in our model, the interactions are 'dense' and 'local', whereas in the models of Vvedenskaya et.al. [30] (the supermarket paradigm) and others, the interactions are 'weak' and 'global'. By 'dense' we mean that the neighboring queues in our present context interact for every customer through impeding each other by an interference of $a_i$. However, two nodes 'separated' in space do not interact with each other at all. Hence, the graph on which the queues are located play an important role in our model. On the other hand, in the super-market type of model, any two queues interact with probability proportional to $1/n$, i.e. there is no graph structure and each arrival interacts with every pair, but any given pair interacts with probability $1/n$. Thus, we view this model as 'weak' and 'global'. However, such an interaction model only makes sense for a finite number of queues, i.e. $n$ must be finite and then the asymptotic considered must be by letting $n$ go to infinity. However, in our case, we do not need this, as there is a natural limiting object ($\mathbb{Z}^d$) on which we can define our dynamics. This is one of our motivations that although mathematically the model is an infinite plane one, it represents the asymptotic of a large wireless network, much like the large-system asymptotic considered in the supermarket type models.

## 3.4 Partial Progress and Future Questions

In this section, we outline some of the results on this model we manage to obtain without giving any proofs due to space constraint.

1. The model can be constructed on a nice probability space.

2. We have the following $0-1$ law $\mathbb{P}[\cap_{i\in\mathbb{Z}^d}\lim_{t\to\infty}x_i(t)<\infty]\in\{0,1\}$. Thus, we call the whole network stable if and only if queue 0 (or any other queue since the model is translation invariant) is stable.

3. If $\lambda<\frac{1}{\sum_{i\in\mathbb{Z}^d}a_i}$, then the system has a at-least one stationary regime.

4. For all $\lambda<\frac{1}{\sum_{i\in\mathbb{Z}^d}a_i}$, if the system were to have to a stationary regime, then it must be the one we found in the step above. Thus, there is an unique stationary distribution. Moreover, the average queue length of queue 0 in steady state is $\frac{\lambda a_0}{1-(\sum_{i\in\mathbb{Z}^d}a_i)\lambda}$. However, it is not clear at this stage whether for every reasonable initial condition, and all $\lambda<\frac{1}{\sum_{i\in\mathbb{Z}^d}a_i}$, we will have weak convergence of the queue lengths. We only know that weak-convergence holds for the empty initial configuration. But for the non-empty case, we have not yet proved convergence to the stationary distribution. There could possibly be a phase-transition here depending on the

The immediate future questions we want to address on this model are the following one.

1. Is it true that if $\lambda>\frac{1}{\sum_{i\in\mathbb{Z}^d}a_i}$, then the system is transient ? In particular is the system also rate-unstable in this regime ?

2. The existence of the stationary distribution we have proven so far is only the minimal solution. In particular, we obtain the minimal solution by starting with an empty

initial configuration and then observing that the queue lengths converge weakly with the mean as that given above. Is it true that for all $\lambda$ and for all reasonable initial conditions, we will have weak convergence of queue lengths ? If the answer is no, then this will indicate the presence of long-range correlations for certain regimes in the model.

# Chapter 4

# Technology Diversity - A Framework for Designing Base-Station Association Policies in Cellular Networks

This chapter is a summary of the published paper [8]. Due to space constraints, we just give a brief description of this paper here and refer the reader to [8] for more details. The reproduction of parts of the paper here are in accordance to IEEE guidelines.

## 4.1 Introduction

In traditionally operated mobile networks, each user (mobile) is obliged to subscribe to a particular operator and has access to the base stations owned by the operator (or to Wi-Fi access points administered by the operator). A new paradigm known as *mobile virtual network operators* (MVNO) is currently reshaping the wireless service industry. The idea is to provide higher service quality and connectivity by pooling and sharing the infrastructure of multiple wireless networks. A recent remarkable entrant such as Google is testing the water in the US market under the name of "Project Fi", whose main feature is improved coverage provided through outsourcing infrastructure from its partners, T-Mobile, Sprint and their Wi-Fi networks. In the meantime, the European Commission has been ruling favorably for MVNOs since 2006, so as to make the European wireless market more competitive [38], thereby facilitating investment in MVNOs in Europe. These virtual operators can take advantage of the hitherto impossibility to cherry-pick different network operators which use separate bandwidths, and even different wireless access technologies, for improvement of user experience. It is reported [39] that the market share of these operators, especially in mature markets, ranged from 10% (UK and USA) to 40% (Germany and Netherlands) as of 2014. However, these unprecedented diversities in terms of bandwidths and wireless technologies raise a challenging question on how to harness them in large-scale wireless networks.

In the rest of the chapter, we use the terminology "technology diversity" to refer to (i) several networks operated on orthogonal bandwidths and (ii) different cellular technologies (e.g. 3G and 4G), both of which can be shared by MVNOs.

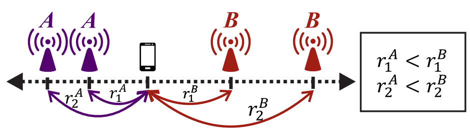


Figure 4.1: Motivational example with two technologies: $r_1^A < r_1^B$ and $r_2^A < r_2^B$.

Notably, the *de facto* standard association policy in existing wireless networks consists in associating each user equipment (UE) with the nearest base station (BS) or access point where one typical aim is to maximize the likelihood of being covered or connected. One of the main points of the present chapter is that this is no longer optimal in these emerging virtual networks, as further discussed in Section 4.1.2. The subtle distinction arising from technology diversity is illustrated in Fig. 4.1, where $r_1^A$ and $r_2^A$ (respectively, $r_1^B$ and $r_2^B$) denote the distances to the nearest and second-nearest BSs of technology $A$ (respectively, technology $B$) from the UE located at the origin. Also, we assume that $r_1^A < r_1^B$, i.e. the nearest BS of technology $A$ is the nearest to the UE, and there are only four BSs as shown in Fig. 4.1, which are identical except that they operate on different technologies (i.e. non-overlapping bandwidths). In the single technology case ($A = B$), the UE can simply associate with the BS at $r_1^A$. However, if $A \neq B$, e.g. the two technologies operate on different bandwidths, the locations of the strongest interferers, $r_2^A$ and $r_2^B$ (the second-nearest BSs), may *overturn* the choice of technology $A$ when the strongest interferer of technology $B$ is much farther from the UE than that of technology $A$, i.e. $r_2^B \gg r_2^A$, thus boosting the signal-to-noise-ratio (SINR) of technology $B$. In light of this example, optimal association in such networks requires new paradigms.

We can further generalize the above example and envisage a practical scenario where each UE can obtain the information about several received pilot signals of nearby BSs, as in 3G and 4G cellular networks, which can be translated into a vector of distances. In this paper, we are interested in investigating the following question.

Q: *How to leverage the diversity brought in by the choice to associate with various non-overlapping networks ? How much should a mobile phone learn about the instantaneous network conditions to exploit the diversity ?*

### 4.1.1 Main Contributions

Our main contribution in this chapter is to formalize the above question into an abstract mathematical framework. We propose a simple abstraction for the network model and quantify the question how much resources should a mobile phone expend in learning about the instantaneous network condition. Secondly, we look at the question of given that the UE has expended its resources to have an accurate knowledge of the instantaneous network conditions, how should it exploit this information to improve performance. To the best of our knowledge, a rigorous study of this problem in the context of Cellular networks has not been undertaken before and our work is the first to lay a rigorous framework for evaluating association policies.

### 4.1.2 Related Work

The policy of associating each UE to the nearest BS or the BS with the strongest received power has been taken for granted in the vast literature on cell association. This is for instance the case in the stochastic geometry model of cellular networks [40]. The rationale is clear. This leads to the highest connectivity for each UE to choose the nearest BS unless it is possible to exploit the time-varying fading information, which is often unavailable in practice. Even with the recent emergence of heterogeneous wireless networks, also called *HetNet*, the rule is still valid in terms of coverage probability. That is, a UE is more likely to be covered if it associates with a BS whose received long-term transmission power (called pilot power) is the strongest. A stochastic geometry model to exploit this heterogeneous transmission powers of BSs belonging to multiple tiers in HetNets along with *fading* information has been investigated in [41].

However, from the perspective of load balancing between cells, the rule is invalid in general because each UE might be better off with a lightly-loaded cell rather than heavily-loaded one irrespective of the distances to them. In particular, in HetNet scenarios, it is important to distribute UEs to macro-cells and micro-cells so that they are equally loaded. The optimal association in the HetNet setting is inherently computationally infeasible, i.e. NP-Hard, whereas the potential gains from load-aware association schemes are much higher [42]. To tackle this problem, a few approximate or heuristic algorithms were proposed based on convex relaxations [42, 43] and non-cooperative and evolutionary games [44, 45]. Most of these algorithms are iterative in nature, requiring many rounds of messaging between UEs and BSs for their convergence.

It must be stressed that the multiple technology setting studied here is a largely unexplored territory where the *validity* of the standard rule to associate with the nearest BS is undermined, which is unprecedented in the literature as exemplified in Fig. 4.1. Lastly, while there have been considerable work adopting stochastic geometry models for analyzing *given* algorithms in large wireless networks, our work is a radical turnaround in the way of harnessing the model: we investigate new opportunities to tailor and design such algorithms to optimize the performance.

## 4.2 Stochastic Network Model

In this paper, we consider adapting association schemes to ameliorate any performance metric in a downlink cellular network that is a function of the SINR received at a single typical UE. To this aim, we first describe a generic stochastic model of the network and define the general performance metric that is induced by an association policy of the UE of interest, which are assumed to be decoupled from those of other UEs.[1] Note that we retain our stochastic network model in the most generic form for easier mathematical manoeuvrability of key results in Section 4.3, which in fact holds for for a large class of point processes (PPs).

### 4.2.1 Network Model

We consider $T$ different technologies where $T$ is finite. The BS locations of technology $i \in [1, T]$ are assumed to be a realization of a homogeneous Poisson-Point Process (PPP) $\phi_i$ on $\mathbb{R}^2$ of intensity $\lambda_i$ independent of other PPPs. The typical user, from whose perspective we perform the analysis, is assumed to be located at the origin, without loss of generality. Denote by $r_j^i \in \mathbb{R}_+$ the distance to the $j$th closest point of $\phi_i$ to the origin, or equivalently the $j$th nearest BS, where ties are resolved arbitrarily. Hence $r_1^i$ denotes the distance to the closest point (BS) of $\phi_i$ from the origin.

Each BS of technology $i$ transmits at a fixed power $P_i$. The received power at a UE from any BS is however affected by fading effects and signal attenuation captured in the propagation model, typically through the path-loss exponent. We assume independent fading, i.e. the collection of fading coefficients $H_j^i$, which denotes the corresponding value from the $j$th nearest BS in technology $i$ to the UE, are jointly independent and identically distributed

[1]Note that extending this framework and results therein to the case where the association policy of a user is affected by those of other users (e.g. load-balancing in HetNet) is mathematically far more challenging and thus is left to future work.

according to some distribution function. We model the propagation path loss through a non-increasing function $l_i(\cdot) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, where $i \in \{1, 2, ..., T\}$, i.e. the propagation model for each technology is determined by a possibly different attenuation function. Hence, the signal power received at the typical UE from the $j$th BS of technology $i$ is $P_i H_j^i l_i(r_j^i)$. For mathematical brevity, we henceforth consider the point process $\phi_i$ of technology $i$ where each point is *marked* with an independent mark denoting the fading coefficient between the point (BS) to the origin (UE). We can assume that all the random variables belong to a single probability space denoted by $(\Omega, \mathcal{F}, \mathbb{P})$ [46].

### 4.2.2 Information at a UE

Another point at issue in this paper is the tradeoff between the cost of "information" available at UE and the performance gain attained by the association policy making use of that information. For easier presentation of results, e.g. Theorem 5, the notion of information is encapsulated in a sigma-field $\mathcal{F}_I$ which is a sub-sigma algebra of the sigma-algebra $\mathcal{F}$ on which the marked point processes $\phi_i$ are defined. A sub-sigma algebra $\mathcal{F}^{'}$ of $\mathcal{F}$ is such that $\mathcal{F}^{'} \subseteq \mathcal{F}$. An example of information is $\mathcal{F}_I = \sigma\left(\cup_{i=1}^{T} \phi_i(B(0, w))\right)$, which corresponds to the sigma-field generated by the point process up to distance $w$ from the origin. In other words, the UE can estimate BS locations of different technologies $r_j^i$ such that $r_j^i \leq w$. However, note that, we use the information sigma algebra $\mathcal{F}_I$ more generally, which could potentially include fading and shadowing and not just the distances as given in the above example.

### 4.2.3 Association Policies

An association policy governs the decisions on which technology and BS the typical user (who is located at the origin) should associate with. More formally, an association policy $\pi$ is a measurable mapping, i.e. $\pi : \Omega \rightarrow [1, T] \times \mathbb{N}$ which is $\mathcal{F}_I$ measurable. As stated before, we assume that all additional random variables needed by the policy $\pi$ are $\mathcal{F}_I$

| Notation | Brief Description |
|---|---|
| $\phi_i$ | Point process corresponding to technology $i \in [1, T]$ |
| $\lambda_i$ | Intensity (density) of point process $\phi_i$ |
| $p_i(\cdot)$ | Performance function when associated with technology $i$ |
| $\mathcal{F}_I$ | Information available at the typical UE |
| $j_i$ | $\arg\sup_{j\geq 1} \mathbb{E}[p_i(\mathrm{SINR}_0^{i,j})\|\mathcal{F}_I]$ |
| $i^*$ | The technology chosen by an association policy |
| $\mathcal{R}_I^{\pi}$ | Average performance of association policy $\pi$ with $\mathcal{F}_I$ |
| $\mathcal{R}_I^{\pi^*}$ | Average performance of optimal policy $\pi^*$ with $\mathcal{F}_I$ |

Table 4.1: Table of Notation

measurable. The interpretation of the policy $\pi$ being $\mathcal{F}_I$ measurable is that a typical UE decides to choose a technology and a BS to associate with based only on the information obtainable in the network. It is important to note that while our discussion in this paper mainly revolves around optimal policies denoted by $\pi^*$, our methodology for the performance evaluation in Section **??** can be applied for any (suboptimal) policy.

### 4.2.4 Performance Metrics

All performance metrics considered in this work are functions of SINR (Signal to Interference plus Noise Ratio) received at the typical UE. The SINR of the signal received at the origin from the $j$th nearest BS of technology $i$ is:

$$\mathrm{SINR}_0^{i,j} = \frac{P_i H_j^i l_i(r_j^i)}{N_0^i + \sum_{k\in\mathbb{N}\setminus\{j\}} P_i H_k^i l_i(r_k^i)},$$

where $N_0^i$ is the thermal noise power which is a fixed constant for each technology $i \in \{1, 2, ..., T\}$. In order to encompass a general set of most useful performance metrics in wireless networks, the performance of different association policies are evaluated through non-decreasing functions of the SINR observed at the typical UE. Formally, let $p_i(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$

be a non-decreasing function for each $i \in \{1, 2, ..., T\}$ which represents the metric of interest if the typical UE associates with technology $i$. Since $\pi$ takes values in two coordinates $[1, T] \times \mathbb{N}$ (Section 4.2.3), we divide them into separate coordinates which are denoted by $\pi(0) \in [1, T]$ and $\pi(1) \in \mathbb{N}$, respectively corresponding to the technology and BS chosen by the policy. Then the performance of the association policy $\pi$ when the information at the typical UE is quantified by $\mathcal{F}_I$ is then given by:

$$\mathcal{R}_I^{\pi} = \mathbb{E}[p_{\pi(0)}(\text{SINR}_0^{\pi})]. \tag{4.1}$$

The subscript $I$ refers to the fact that the information present at the typical UE is $\mathcal{F}_I$. The performance metric $\mathcal{R}_I^{\pi}$ is averaged over all realizations of the BS deployments, fading variables, and any additional random variables used in the policy $\pi$.

Two well-known examples of performance metrics used in practice are coverage probability and average achievable rate. Coverage probability corresponds to setting the function $p_i(x) = \mathbf{1}(x \geq \beta_i)$, which is the chance that the SINR observed at a UE from technology $i$ exceeds a threshold $\beta_i$. The other common performance metric of interest, average achievable rate, is defined as $p_i(x) = B_i \log_2(1 + x)$, where the parameter $B_i$ is the bandwidth of technology $i$. All results on optimal association policy and performance evaluation are stated on the assumption of a general function $p_i(x)$.

## 4.3 Optimal Association Policy

The optimal association policy denoted by $\pi^*$ is

$$\pi_I^* = \arg\sup_{\pi} \mathcal{R}_I^{\pi}, \tag{4.2}$$

where the supremum is over all $\mathcal{F}_I$ measurable policies. From a practical point of view, the optimal association policy is the one that maximizes the performance of the typical UE among all policies having the same "information". In this setup of optimal association, however, we always assume that the typical UE has knowledge of the densities $\lambda_i$ of the different technologies and the fact that they are independent PPPs although several fundamental results can be easily extended to more general point processes.

Since, we are interested in maximizing an increasing function of the SINR of the typical UE, the optimal association rule is clearly to pick the pair of technology and BS which yields the highest performance conditional on $\mathcal{F}_I$.

**Proposition 1.** *The optimal association algorithm when the information at the typical UE is given by the filtration $\mathcal{F}_I$ is such that*

$$\begin{aligned} \pi_I^*(0) &= \arg\max_{i\in[1,T]}\sup_{j\geq 1}\mathbb{E}[p_i(SINR_0^{i,j})|\mathcal{F}_I], \\ \pi_I^*(1) &= \arg\sup_{j\geq 1}\max_{i\in[1,T]}\mathbb{E}[p_i(SINR_0^{i,j})|\mathcal{F}_I], \end{aligned} \tag{4.3}$$

*where the UE must pick the technology $\pi_I^*(0)$ and the $\pi_I^*(1)$-th nearest BS to the origin in $\phi_{\pi_I^*(0)}$.*

The performance of the optimal association is

$$\mathcal{R}_I^{\pi^*} = \mathbb{E}[\sup_{j\geq 1}\max_{i\in[1,T]}\mathbb{E}[p_i(\mathrm{SINR}_0^{i,j})|\mathcal{F}_I]]. \tag{4.4}$$

Since $[1,T]$ and $\mathbb{N}$ are countable sets, the order of the maxima in (4.4) does not matter. An important point to observe is that the optimal association given in (4.3) depends on the choice of the performance metric $\{p_i(\cdot)\}_{i=1}^T$. Hence, the optimal association rule would be potentially different if one was interested in maximizing coverage probability as opposed to maximizing rate-related metrics for instance.

### 4.3.1 Ordering of the Performance of the Optimal Association

In this sub-section, we prove an intuitive theorem (Theorem 5) stating that "more" information leads to better performance.

**Theorem 5.** *If* $\mathcal{F}_{I_1} \subseteq \mathcal{F}_{I_2}$*, then* $\mathcal{R}_{I_1}^{\pi^*} \leq \mathcal{R}_{I_2}^{\pi^*}$ *where the association rule is the optimal one given in* (4.3)*.*

This theorem establishes a partial order on the performance of the optimal policy under different information scenarios at the UE for any performance functions $\{p_i(x)\}_{i=1}^{T}$.

### 4.3.2 Optimal Association in the Absence of Fading Knowledge

The following lemma is quite intuitive and affirms that the optimal strategy for a UE in the absence of fading knowledge is to associate to the nearest BS of the optimal technology.

**Lemma 1.** *If the information* $\mathcal{F}_I$ *at the typical UE does* ***not*** *contain the fading random variables, then* $j_i = \arg\sup_{j\geq 1} \mathbb{E}[p_i(SINR_0^{i,j})|\mathcal{F}_I] = 1$ *and hence* $\pi_I^*(1) = j_{\pi_I^*(0)} = 1$*.*

## 4.4 Max-Ratio Association Policy

While the parametric framework in Section 4.3 paves the way for designing the association policy maximizing various metrics, the optimal schemes encapsulated in (4.2) and (4.3) are amenable to tractable analysis only with the knowledge about the underlying PPPs $\phi_i$, i.e. their intensities $\lambda_i$. On the other hand, it is less conventional at the present time, if not unrealistic, to assume that the densities $\lambda_i$ are available at the UE in a real network. More importantly, in certain deployment scenarios, it is highly likely that the BS distribution follows a non-homogeneous point process with density (intensity) varying with the location over the network, thereby invalidating the homogeneous PPP assumption.

From the computational perspective, the optimal association can often demand substantial processing power of the UE particularly when the resulting association tailored for a specified performance metric is not simplified into a tractable closed-form expression. In this light, it is desirable to have policies that are completely oblivious to any statistical modeling assumption on the network, i.e. minimalistic policies exploiting universally available information such as distances to BSs, which can be computed from received pilot signal powers in 3G and 4G networks. To address these issues, we propose a *max-ratio* association policy. This policy has access to the ratio $r_2^i/r_1^i$ information for each technology $i$, i.e. the information $\mathcal{F}_I = \sigma\left(\cup_{i=1}^T r_2^i/r_1^i\right)$. The max-ratio association is formally described by

$$i^* = \max_{i \in [1,T]} r_2^i/r_1^i,\ j^* = 1.$$

This ratio maximization implies that we place a high priority on a technology where simultaneously the distance to the nearest BS $r_1^i$ is smaller and that to the second-nearest BS $r_2^i$ is larger than other technologies. Note also that the above expression can be easily rearranged into the ratio of the received pilot powers of the nearest and second-nearest BSs when the BS transmission powers within each technology is the same. We show in Theorem 6 that although this policy *per se* is a suboptimal heuristic, it is optimal (in the sense of (4.3)) under a certain limiting regime of the wireless environment.

**Theorem 6.** *Let the noise powers $N_0^i = 0$ for all technologies $i$ and the performance function for all technologies $p_i(\cdot) = p(\cdot)$ for all $i$. Consider the family of* ***power-law*** *path-loss functions $\{l^{(\alpha)}(\cdot)\}_{\alpha>2}$ where $l^{(\alpha)}(x) = x^{-\alpha}$. Let $k$ be* ***any*** *integer greater than or equal to* 2*. If the information at the UE is the $k$-tuple of the nearest distances of each technology $i$ i.e. $\mathcal{F}_I = \sigma(\cup_{i=1}^T (r_1^i \cdots, r_k^i))$, then*

$$\pi_\alpha^*(0) \xrightarrow{\alpha\to\infty} \arg\max_{i \in [1,T]} \frac{r_2^i}{r_1^i}\ \ a.s., \tag{4.5}$$

*where $\pi_\alpha^*$ is the optimal association as stated in* (4.3)*. Recall $\pi_\alpha^*(1) = 1$, $\forall \alpha$ from Lemma 1.*

This theorem states that max-ratio association is optimal in cases where the signal is drastically attenuated (i.e. large path-loss exponents) with distance, e.g., *metropolitan* or *indoor* environments where the exponent reach values higher than 4, e.g. $\alpha \in [4, 7]$. It is noteworthy that $\alpha$ at higher frequencies as in LTE networks tends to be higher (See, e.g. [**?**, Chapter 2.6] and references therein). In addition, another remarkable implication of this theorem is that it suffices for the asymptotic optimality to exploit the *reduced* information $r_1^i/r_2^i$ per technology in lieu of the given original information, i.e. $r_1^i$ and $r_2^i$. Also, any supplementary information on distances (or received pilot powers) to the third-nearest or farther BSs is *superfluous* and does not influence the optimality of the association.

## 4.5 Simulations and Numerical Results

In this section, we provide more insights into our framework and results by performing simulations and noticing their trends. In performing the simulations, we take as performance metrics, the coverage probability with $p_i(x) = \mathbf{1}(x \geq \beta_i)$ and the average rate with $p_i(x) = \log_2(1 + x)$.

### 4.5.1 Comparison of Schemes and Technology Diversity

The first two graphs in Fig. 4.2 compare the coverage probability of various association schemes with path-loss exponent $\alpha = 4$ for different number of technologies, $T = 5$ and $T = 8$. We observe in all graphs that the Max-Ratio association scheme outperforms the optimal association policy under the case when only the nearest BS distances are known. More importantly, the Max-ratio association performs *almost as well* as the optimal association under the knowledge of nearest 2 BSs per technology for this typical value of path-loss exponent, not to mention that it outperforms the nearest BS association significantly, particularly when the technology diversity is higher, i.e. $T = 8$.

The rightmost graph in Fig. 4.2 depicts the average achievable rate for path-loss exponents $\alpha \in [2.5, 7]$, which empirically corroborates the statement of Theorem 6 that Max-Ratio is the optimal policy when nearest $k \geq 2$ BS per technology are known in the high path-loss regime. Remarkably, Max-Ratio and the optimal association with two nearest

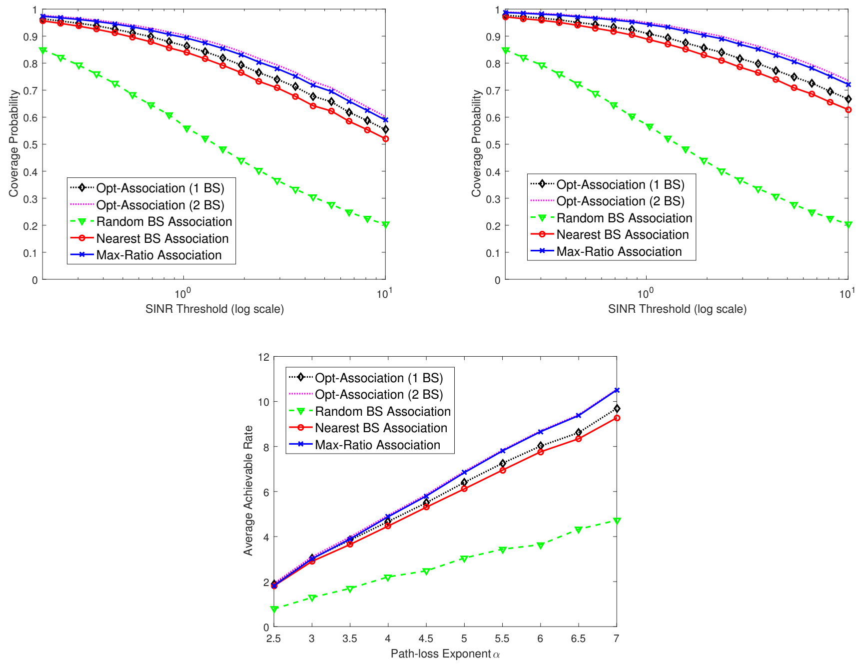


Figure 4.2: Comparison of various association schemes. The first two graphs on the left compare coverage probability where $l_i(r) = r^{-4}$ and $T = 5$ for the first figure and $T = 8$ for the second figure. The rightmost graph compares average achievable rate where $\alpha$ is varied on the x-axis and $l_i(r) = r^{-\alpha}$.

BS distances performs almost equally (indistinguishable in the graph) for $\alpha \geq 5$. That is, a simple non-parametric policy like the max-ratio performs as well as the optimal association policy in which the entire network topology is known (the best possible performance) even in the finite path-loss case. It is also noted that the random BS association, which is the only policy oblivious to technology diversity, results in poor performance in all cases. Thus it is beneficial for MVNOs to leverage the technology diversity in any possible manner by all means.

## 4.6 Conclusions

In this work, we explored the potential to boost the service performance of wireless networks without incurring additional infrastructure cost by capitalizing on a new form of diversity, which can be either several networks operated on orthogonal bandwidths or multiple wireless technologies pooled by some mobile virtual network operators. We proposed a generic stochastic geometry model for designing association policies that can optimize any desired performance metric. We characterized the optimal policies by establishing a natural monotonicity with respect to increasing information, thereby giving a partial order on the performance without explicit computations. We also proposed a pragmatic data-dependent association policy and showed it to be optimal under reasonable assymptotics. In simulations, we see that the practically reasonable finite network parameters mimic well the limit in our theoretical result, thereby giving great validity to our heuristic association scheme. We believe that our heuristic scheme can serve as an alternative to the standard rules in urban or metropolitan environments with severe signal attenuation which better exploits the new form of diversity.

# Chapter 5

# Community Detection on a Spatial Random Graph

This is proposed future work. We outline the model and motivations and describe some partial results. We also list some open questions on which we can hopefully make progress in the final thesis.

## 5.1 Introduction and Related Work

In this work, we study the problem of community detection on graphs. The problem of community detection is the task of partitioning an underlying population into groups such that members of a group are 'similar' and members across groups are 'dis-similar'. This problem is non-trivial in practice since the notions of group membership is often not precise, and even when precisely defined, group membership information is only known indirectly and need to be estimated. In this chapter, we restrict ourselves to a sub-class of this problem of when the notion of group membership is precise, but information on group membership is known indirectly only through a labeled graph with the population as the vertex set. This sub-class although seems mathematically restrictive, is still rich enough to encompass many interesting applications (see [7] and the references therein).

The most popular model for studying graph clustering is through a random graph model known as the Stochastic Block Model or the Planted Partition Model. In its simplest version, this model is a multi-type Erdős- Rényi random graph model. The underlying population is assumed to be partitioned into different groups corresponding to the ground truth of partition. Conditional on the partition, any two members are connected by an edge independently of every other pair of edges with a probability that depends on the group membership of the end-points. If the probabilities of connecting are different enough for different types of pairs of end-points, one would hope that a single random instance of the graph on this population will contain information about the underlying group membership of the nodes. Indeed, the task of community detection then is to infer the underlying partition of the population from a single sample of a random graph drawn by assuming the partition. This model has received a lot of attention lately ( [33], [34], [35]) which have provided rigorous phase-transitions which provide benchmarks for algorithm designers.

However, from an application point of view, the SBM suffers from some obvious shortcomings. Most notably, if the graph is forced to be 'sparse', then it is well known that the SBM is 'locally tree-like' (see [36] for a precise statement), in the sense with high probability, a random sample from a typical vertex will 'look' like a tree. However, it is well known that real social networks exhibit a form of 'transitivity', i.e. if $i$ and $j$ are friends and $j$ and $k$ are friends, then it is more likely that $i$ and $k$ are friends ( [3]). Structurally, one expects to see a lot of triangles in a social graph, which the SBM can be provably shown not to contain. These observations are classical and have resulted in the development of what are known as 'Latent-Space Models' ( [3], [4]). In these models, the agents of a social network are assumed to be embedded in an abstract social space. This space could be any topological group (such as Euclidean Space, the Sphere, the group of automorphisms of some labeled graph ) endowed with a metric. Conditional on the embedding, edges appear between nodes independently across pairs, but with a probability that is non-increasing with distance with respect to the underlying metric. However, for concreteness and as a first starting step, we focus on the case when the embedding of the agents in the social space is a $d$-diemnsional Euclidean space with the topology generated by the Euclidean distance between points. In the future, we plan on generalizing this to other abstract groups.

## 5.2 Mathematical Problem Statement

In this section, we identify the simplest non-trivial mathematical problem of community detection on a random graph drawn from the latent-space model class. We propose to study this problem first before considering a more general version.

### 5.2.1 Random Graph Model

We model the locations of the agents in the social space to be distributed as a homogeneous Poisson Point Process $\phi$ of intensity $\lambda$ on $d$ dimensional Euclidean space. Each agent is randomly and uniformly assigned a community label of either $+1$ or $-1$. The graph is further parametrized by two functions $f_{in}(\cdot), f_{out}(\cdot) : \mathbb{R}_+ \rightarrow [0,1]$ such that for all $r \geq 0, f_{in}(r) \geq f_{out}(r)$. The graph $G$ is drawn on the vertex set $\phi$ where any two points are connected by an edge independently of every other pair of edges conditional on the location and the community labels. Two points at locations $x$ and $y$ in $\mathbb{R}^d$ are connected by an edge in $G$ with probability $f_{in}(||x-y||)$ if they belong to the same community or with probability $f_{out}(||x-y||)$ if they belong to opposite communities. This is a natural 'planted' generalization the classical Random Connection Model (see [9]). We also refer the reader to [9] for a precise construction of this random graph.

### 5.2.2 Community Detection Definition

The notion of community detection we consider is slightly different than what one would expect. We ask for recovery of the underlying partition of the population given the noisy graph process **and** the exact spatial embedding of the nodes in the social space. In practice this is restrictive since one would not know the spatial embedding and it needs to be inferred. In most cases, at-best a noisy positions of the agents in the social space can be known. We realize this obvious shortcoming, but nonetheless consider exact spatial-locations as a starting point, since this in itself presents a formidable mathematical challenge.

To define the problem, we have another technical hurdle in the model. Since our model is a stationary random graph with an infinite population, we will have to partition this infinite population into two stationary sets of inifinite cardinality and compare them. Thus, a natural measure of closeness of partitions as used in prior work on the SBM model will need some care to extend to our setting where the vertex set has infinite cardinality. We bypass this technicality by a slight change in the definition of the problem without losing its essence.

To define the problem, we move to the Palm distribution space of the process $\phi$, i.e. assume that there is a typical vertex of the graph $G$ located at the origin of $\mathbb{R}^d$. Let $(B_n)_{n\in\mathbb{N}}$ be a sequence of compact sets of $\mathbb{R}^d$ such that $B_n \subseteq B_{n+1}$, $0 \in B_0$ and $\cup_{n\in\mathbb{N}} B_n = \mathbb{R}^d$. At the moment, we assume we have such an arbitrary collection and define the problem with respect to this collection of sets. Let $\delta \in (0, 1]$ be arbitrary. Denote by $G_n^{(\delta)}$ to be the graph $G$ with each of its vertices given a label in $\{-1, 0, +1\}$ as follows. If a point of $x \in \phi$ is such that $x \in B_n$, then its label is 0. If a point of $x$ is such that $x \in B_n^{\complement}$, then with probability $1 - \delta$, its label is 0 and with probability $\delta$, its label is its true community label. The randomness in the labeling process is independent across points . In other words, all true community labels of points in $B_n^{\complement}$ are revealed with probability $\delta$ independently.

**Definition 2.** *The community Detection problem for the parameter* $\lambda, f_{in}(\cdot), f_{out}(\cdot), d$ *is said to be* ***solvable*** *if there exists a family of* $\{-1, +1\}$ *valued random variables* $\{\tau_n^{(\delta)}\}$ *such that* $\tau_n^{(\delta)} \in \sigma((\bar{G}_n^{(\delta)}), \bar{\phi}_n^{(\delta)})$ *and an* $\epsilon > 0$ *satisfying*

$$\liminf_{\delta\to 0} \liminf_{n\to\infty} \mathbb{P}_{\phi}^{0}[\tau_n^{(\delta)} = Z_0] \geq \frac{1}{2} + \epsilon \tag{5.1}$$

Our problem can be interpreted as asking whether can one learn something about the label of the typical point at the origin, given very little information at infinity ? It is very straightforward to show that the above definition does not depend on the sets $(B_n)_{n \in \mathbb{N}}$, i.e. if one can solve the problem with respect to some increasing compact sets increasing to the whole plane, then one can solve with respect to any such sequence of compact sets. Hence, we do not mention the sets $B_n$ explicitly in the definition. By analogy to what is considered in the traditional SBM, one would expect that a natural question be that can one partition the nodes of a graph in a large finite ball around the origin into respective clusters such that the fraction of misclassified nodes is strictly smaller than a half as the radius of the ball goes to infinity. We believe that these are essentially equivalent although we cannot prove it. Indeed, our algorithm and the lower bound for this problem is to first partition the unlabeled data and only looks at the revealed labels to break the symmetry and report whether the tagged typical vertex is in community of $+1$ or $-1$.

## 5.3 Partial Progress and Future Work

We have the following preliminary results on this problem thus far.

1. For any fixed $f_{in}(\cdot), f_{out}(\cdot)$ and $d$, the optimal probability of detection is non-decreasing in $\lambda$. Thus, $\lambda$ is a form of Signal-to-Noise ratio in this model and it naturally establishes that there be a phase-transition in $\lambda$ between in-feasibility and feasibility of solving the problem.

2. For any $f_{in}(\cdot), f_{out}(\cdot)$ and $\delta > 0$, we have a lower bound based on the structural properties of the random connection model graph with connection probability $f_{in}(\cdot) - f_{out}(\cdot)$. In particular, we can show that if a random connection model graph on PPP of intensity $\lambda$ in $d$ dimensions with connection function $f_{in}(\cdot) - f_{out}(\cdot)$ does not percolate, then community detection is impossible. This result already gives an indication as to how 'far' must the two connection probabilities be to have any hope of detection. We derive this lower bound by an appropriate 'random-cluster representation' of our

model. As a corollary, we also see that if the average degrees are finite, then exact recovery is impossible, i.e. $\epsilon = \frac{1}{2}$ in Definition 2 can never be achieved.

3. We have a sufficient condition giving that for every $d$ and $f_{in}(\cdot)$ and $f_{out}(\cdot)$ such that $\{r \in \mathbb{R}_+ : f_{in}(r) > f_{out}(r)\}$ has positive Lebesgue measure, there exists a sufficiently high $\lambda_0$ such that for all $\lambda \geq \lambda_0$, our algorithm solves community detection. The algorithm is based on a coupling connection to site percolation.

However, all our results are sub-optimal, in the following sense.

1. There exists an instance of the problem for which our lower bound is provably sub-optimal.
2. There exists an instance of the problem for which our algorithm (or upper bound) is provably sub-optimal.

Both these instances are in the case of infinite average degree, where there are many classical results from Shiryaev [37] that prove our lower and upper bounds are not optimal. In the final thesis, we aim to address these points and hope to make some progress on understanding what is the fundamental bottleneck in performing community detection on spatial random graphs.

We also propose to conduct empirical studies to understand and validate some aspects of our model with real social network data. In particular, we want to understand how the information about an appropriate embedding in a latent social space can aid in community detection.

# Chapter 6

# Coursework

Table 6.1: **Major Coursework**

| **Course Number** | **Course Name** | **Instructor** | **Grade** |
|---|---|---|---|
| EE 381J | Probability & Stochastic Processes I | G. de Veciana | A |
| CS 388R | Randomized Algorithms | G. Plaxton | A |
| EE 381V | Analy. and. Desgn of Comm Ntwks | G. de Veciana | A |
| EE 381K | Advanced Telecom Networks | F. Baccelli | A |
| EE 381V | Large Scale Optimization | S. Sanghavi | A |
| EE 381M | Probability and Stochastics 2 | F. Baccelli | A |
| EE 381K | Wireless Communications | J. Andrews | B+ |
| EE 381V | Advanced Probability: learning/inference/networks | S. Shakkottai | A |
| M 393C | Random Graphs and Stoch. Geom | F. Baccelli | A |

Table 6.2: **Supporting Coursework**

| **Course Number** | **Course Name** | **Instructor** | **Grade** |
|---|---|---|---|
| CSE 384K | Theory of Probability I | G. Zitkovic | A |
| M 394C | Markov Chains and Mixing Times | J. Neeman | A |
| CSE 385R | Real Analysis | L. Bowen | A |
| M 394C | Stochastic Processes | T. Zariphopoulou | A- |
| M 380C | Algebra I | L. Bowen | A- |

# Chapter 7

# Publications

## Publications Related to this Proposal

- A. Sankararaman, F. Baccelli, "Spatial Birth-Death Wireless Networks", *IEEE Transactions on Information Theory*, 2017.

- A. Sankararaman, J. W. Cho and F. Baccelli, "Performance-Oriented Association in Large Cellular Networks with Technology Diversity," *2016 28th International Teletraffic Congress (ITC 28)*, Würzburg, Germany, 2016, pp. 94-102.

## Other Publication

- A. Sankararaman and F. Baccelli, "CSMA k-SIC A class of distributed MAC protocols and their performance evaluation," *2015 IEEE Conference on Computer Communications (INFOCOM)*, Kowloon, 2015, pp. 2002-2010.

## In Preparation

- "Queues on $\mathbb{Z}^d$ with interacting Service Rates", with Sergey Foss and François Baccelli.

- "Community Detection on an Euclidean Random Graph" with François Baccelli.

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# Vita

Abishek Sankararaman received a B.S. and M.S. in Electrical Engineering from Indian Institute of Technology Madras, Chennai, India in 2013. He has been working towards his Ph.D. in Electrical Engineering at The University of Texas at Austin since September 2013. His research interests are broadly in stochastic modeling, analysis and algorithm design for networks. He has held a summer internship with Huawei Research, Santa-Clara, CA as a Data-Scientist in Summer of 2015.

Permanent address: 1626 W 6th street, Apt. A
Austin, TX 78703.

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