Performance Oriented Association in Cellular Networks with Technology Diversity

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Outline

- Motivation and Background.
- Main mathematical model.
- Summary of Results.
- Conclusions

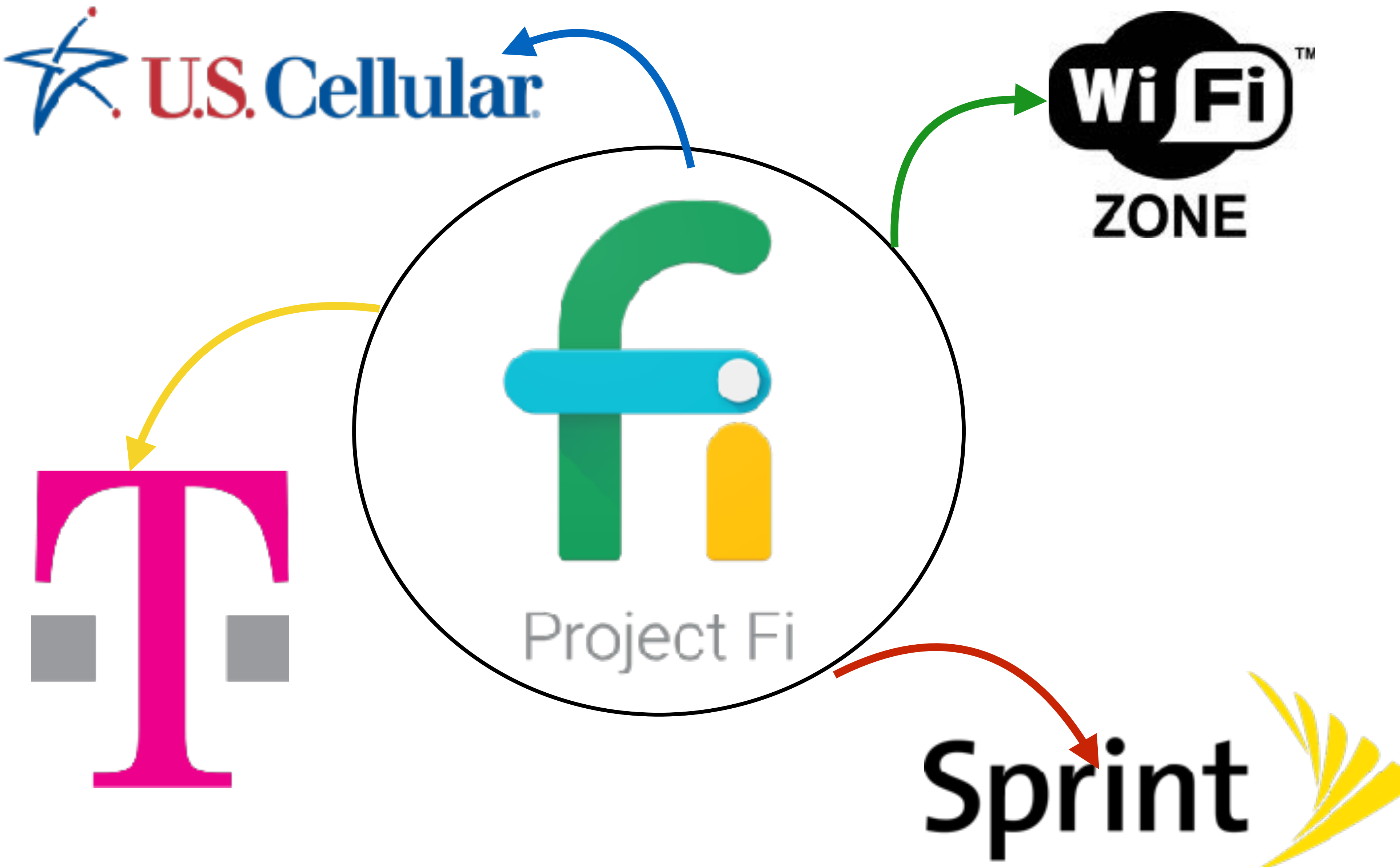
Background

MVNOs (Mobile Virtual Network Operators)

These operators pool together various wireless technologies (for eg. 2G, 3G, 4G LTE) to create a service.

A UE can dynamically chose between the various technologies depending on whichever yields higher instantaneous benefit.

Background



A new model of cellular service by Google

What the Paper is About ? Technology Diversity

Leveraging the presence and *control* of multiple wireless technologies operating on *non-overlapping bandwidth*.

Google Fi - The different cellular operators and WiFi operate on separate bands.

MVNOs - Multiple orthogonal technologies (For ex. 3G and 4G LTE).

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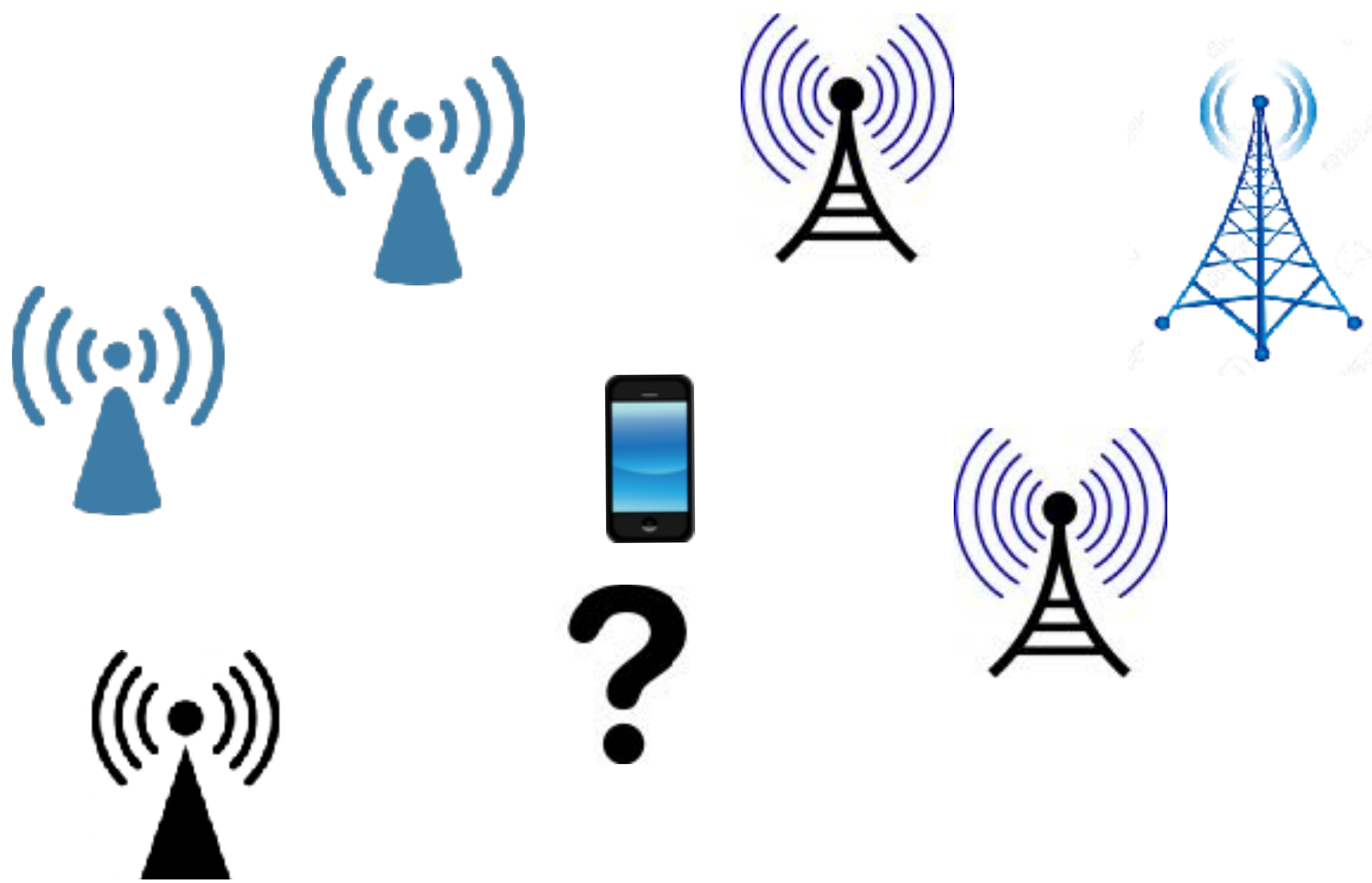
Technology Diversity - *A framework to leverage and evaluate the benefits from the diversity of wireless technologies.*

The Problem we study - Base Station Association

Which Base-Station/Access Point must a UE associate with ?



A principled way to exploit the ***diversity*** in the network.

The “optimal” BS Association is not an obvious choice

First Guess - Connect to the nearest BS irrespective of the type of BS it is.

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Nearest BS

$$\text{SINR} = \frac{\text{Signal}}{\text{Interference + Noise}}$$

SNR  > SNR  (*Basis for nearest BS association.*)

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Thus, in this example SINR  < SINR 

Mathematical Framework - Network Model

Consider a network comprised of T different technologies.

The BS/APs of technology i is distributed as a independent Poisson Point Process $\phi_i \subset \mathbb{R}^2$ with intensity λ_i

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There is a *typical user* at the origin of the Euclidean plane who wishes to associate to a BS.

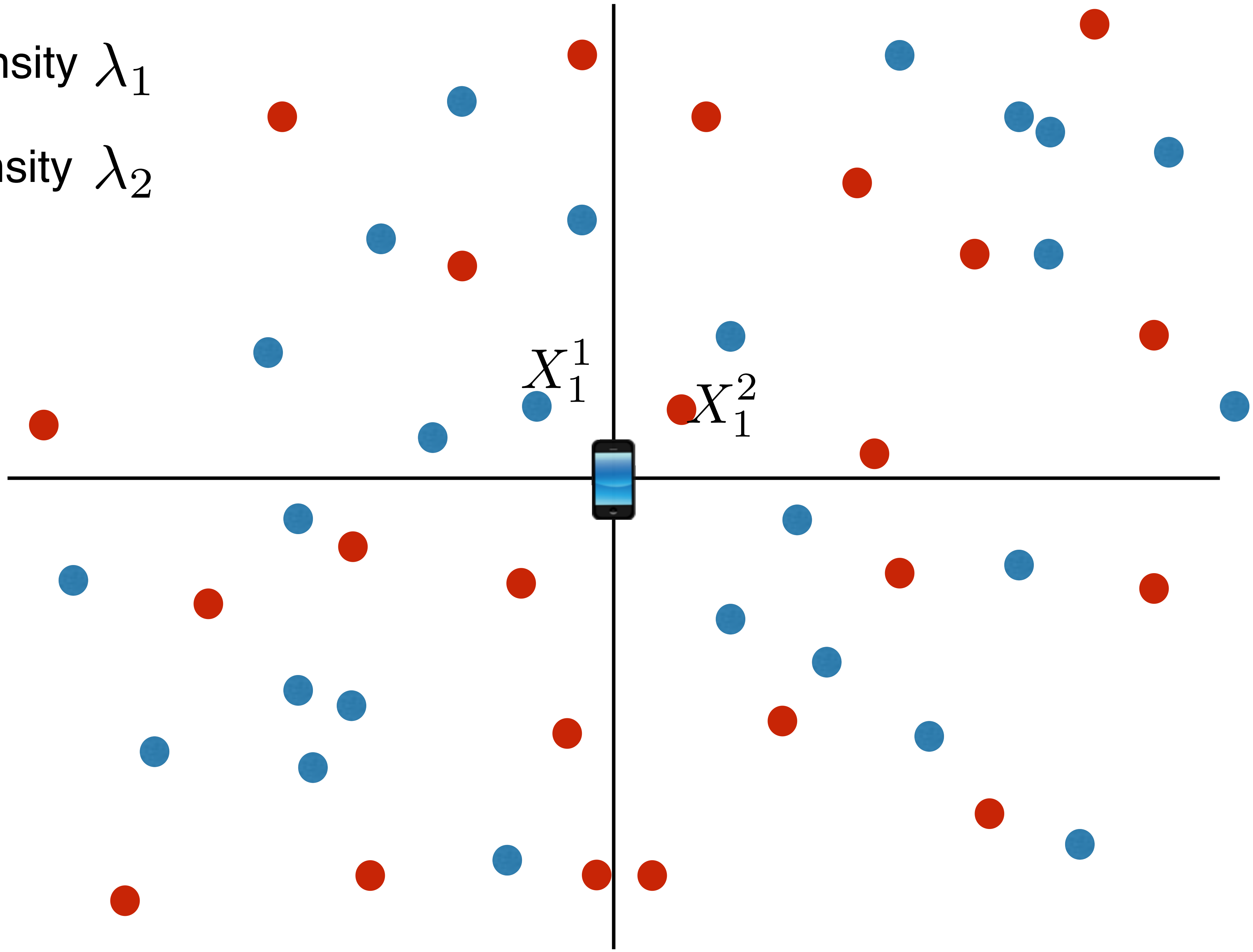
Palm theory connects the viewpoint of a single user to the ***average*** performance experienced by the users in the network.

Mathematical Framework - Network Model

$T = 2$

$\phi_1 =$ ● with intensity λ_1

$\phi_2 =$ ● with intensity λ_2



Mathematical Framework - Signal Model

BS of technology i transmits at power P_i

Signal from BS of technology i attenuated with distance as given by the function $l_i(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$

Independent fading from the jth nearest BS of technology i to the typical user - H_j^i

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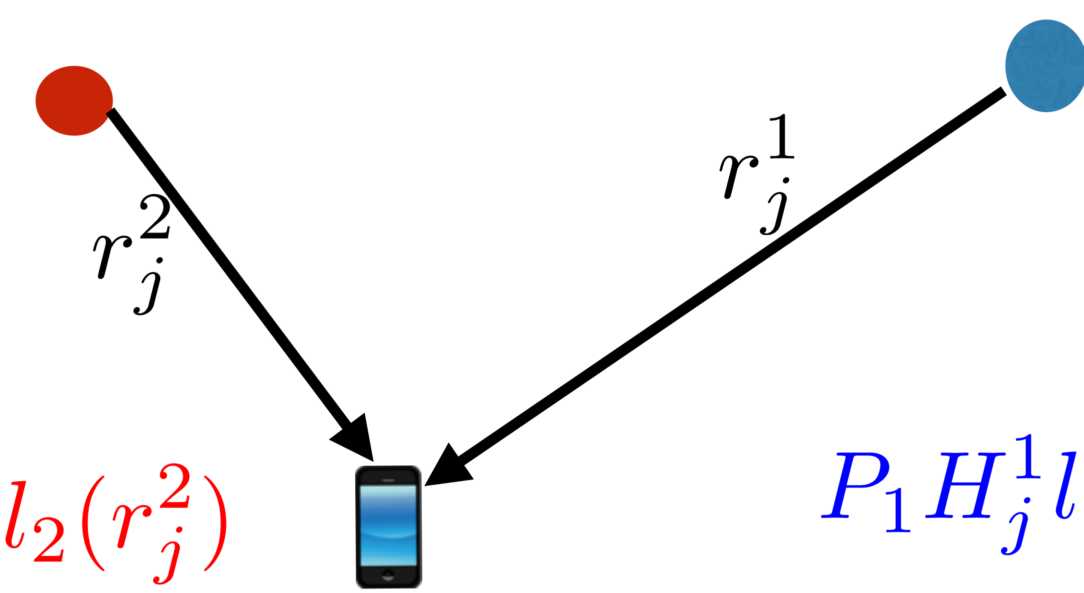

$P_2 H_j^2 l_2(r_j^2)$

(Signal Power from the ● BS)

$P_1 H_j^1 l_2(r_j^1)$

(Signal Power from the ● BS)

Performance Metrics

(Non-overlapping bandwidths $\Longrightarrow$ Interference from only one technology)

$SINR_0^{i,j}$ SINR of the typical UE when it associates to the jth nearest BS of technology i.

- For each technology i, denote by bounded non-increasing functions $p_i(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ denoting the ***reward*** function.

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- If the typical UE connects to the jth nearest BS of technology i, then it receives a ***reward*** of $p_i(SINR_0^{i,j})$

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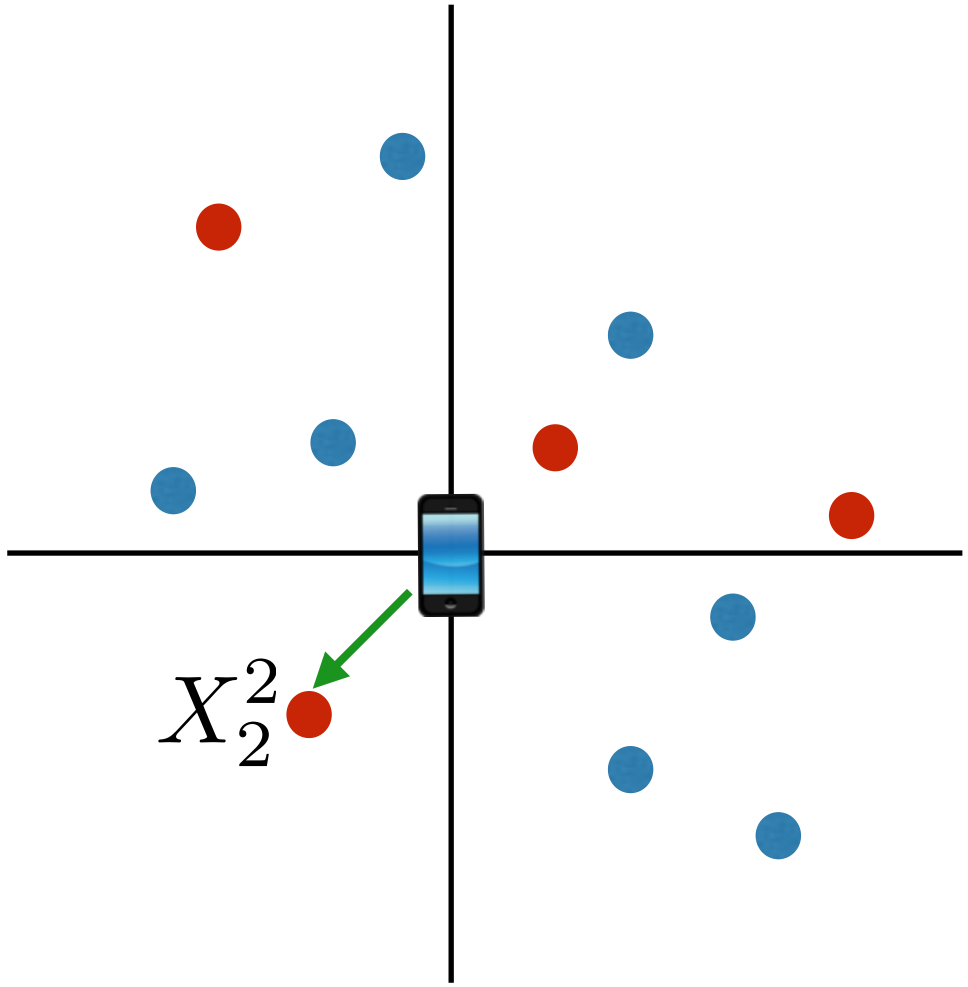

The reward received by the UE in this example is $p_2(SINR_0^{2,2})$

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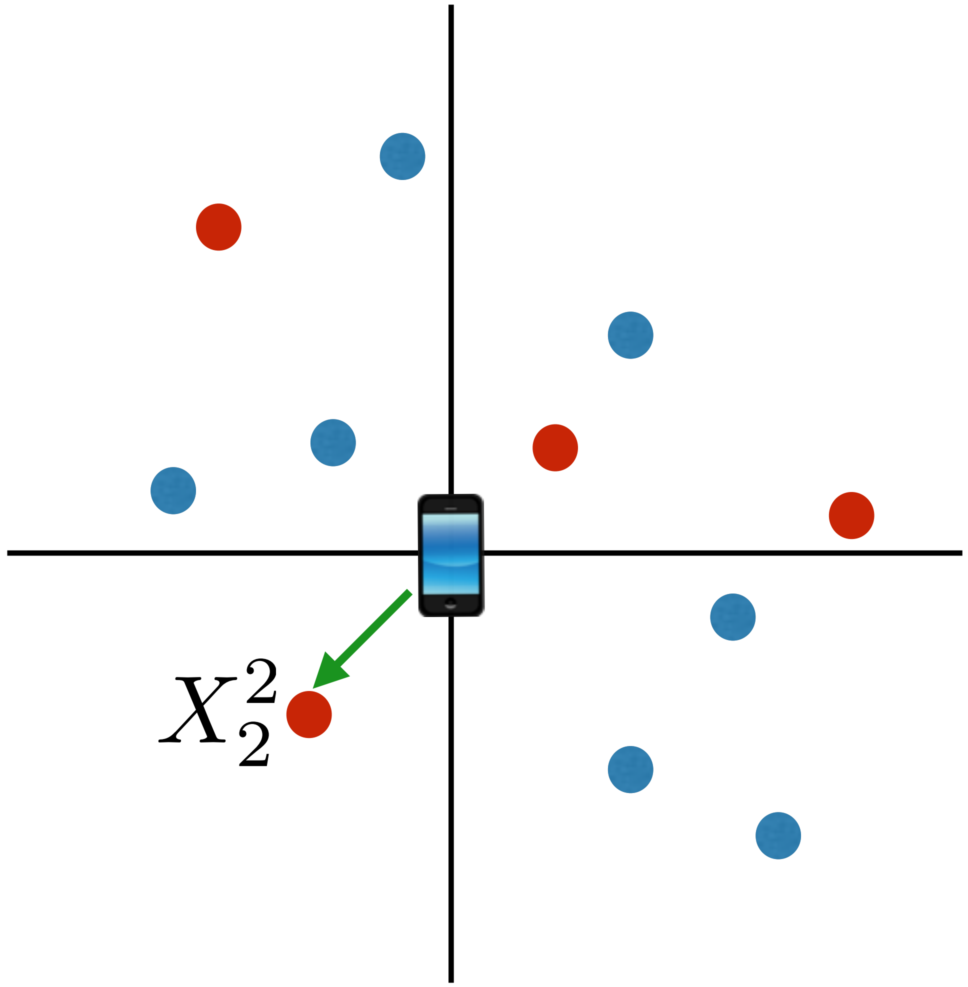

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Examples of common reward functions

- Coverage $p_i(x) = \mathbf{1}(x \geq \beta_i)$
- Average Achievable Rate $p_i(x) = B_i \log_2(1 + x)$

Information at the UE

Goal- Design association schemes exploiting available network ***"information"*** at the UE, that maximize expected reward of a typical UE.

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- Nearest BS of all technologies.
- Nearest k BS of all technologies.
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- Noisy estimate of the instant fading from the nearest k BS of each technology.

How Information affects Optimal Association

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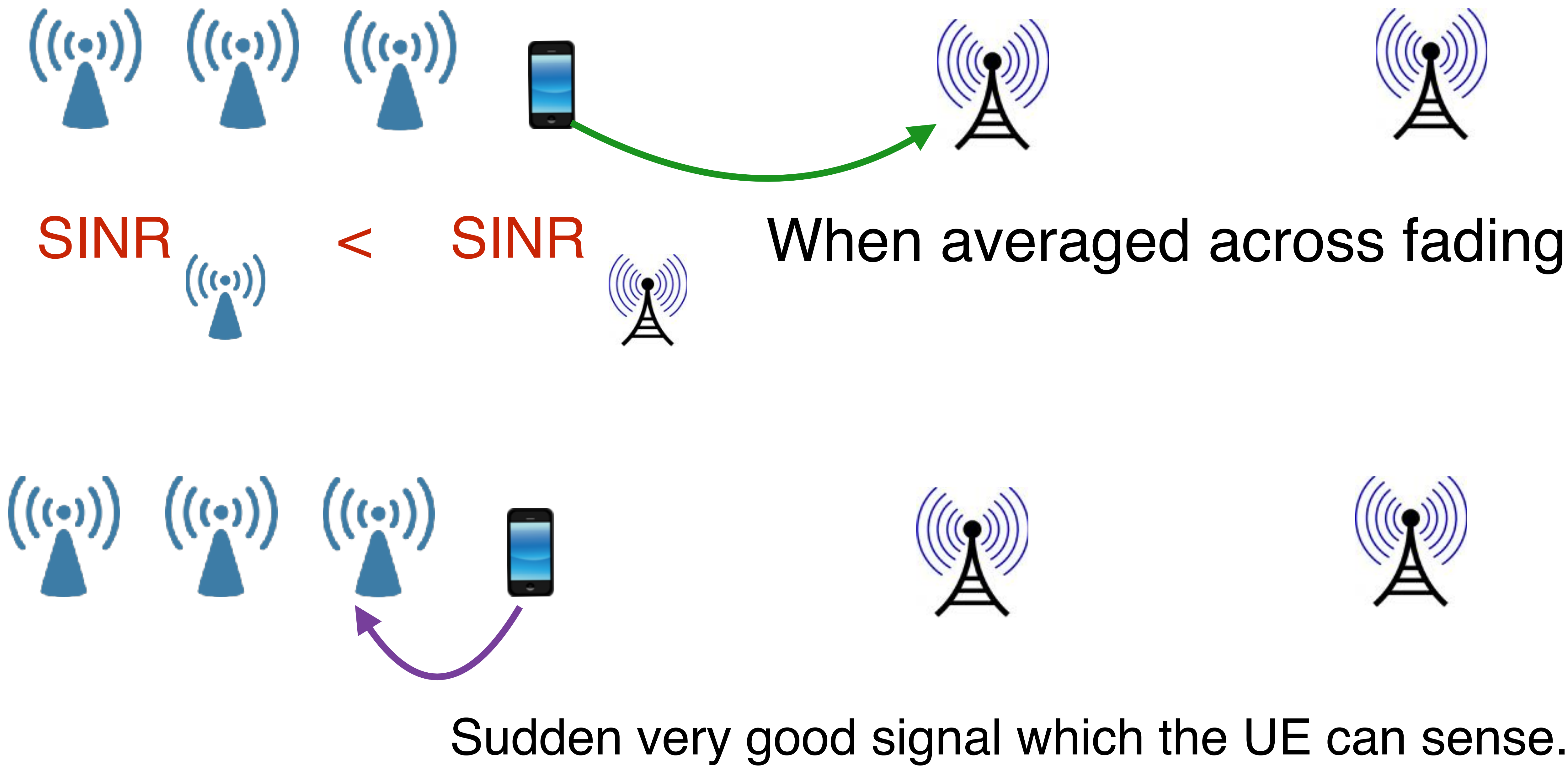

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Sudden very good signal which the UE can sense.

In this case, UE should associate to 

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Notion of Information at the typical UE formalized through the notion of ***filtrations*** of a sigma algebra.

Information at the UE - Formalization

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- Noisy estimate of the instant fading from the nearest k BS of each technology.

$\mathcal{F}_I = \sigma\left(\sigma(\{r_k^i\}_{i=1}^T) \cup \mathcal{F}^{'}\right)$ where $\mathcal{F}^{'} \subseteq \sigma\left(\{H_j^i\}_{j\in[1,k],i\in[1,T]}\right)$

Performance Metric for Association

Recall -

$(\Omega, \mathcal{F}, \mathbb{P})$ The Probability space containing the independent RVs $\{\phi_i\}_{1=1}^{T}$ and $\{H_j^i\}_{i \in [1,T], j \in \mathbb{N}}$

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Performance of policy π is then $\mathcal{R}_\pi = \mathbb{E}[p_{\pi(0)}(SINR_0^\pi)]$

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Proposition:

$$\pi_I^* = \arg \sup_{i \in [1,T], j \in \mathbb{N}} \mathbb{E}[p_i(SINR_0^{i,j}) | \mathcal{F}_I] \quad \text{a.s.}$$

Optimal Association - Comparison of Information

The optimal association π_I^* under information $\mathcal{F}_I$ is given by

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Comparison of schemes without messy computation !

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$\blacksquare$

Optimal Association - Associate to the Nearest BS

__Lemma:__

If the information $\mathcal{F}_I$ is independent of $\sigma\left(\{H_j^i\}_{j\geq 1, i\in[1,T]}\right)$ then, $\arg\sup_{j\geq 1} \mathbb{E}[p_i(SINR_0^{i,j})|\mathcal{F}_I] = 1$ for all i .

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<u>Lemma:</u>

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*In particular, in the **absence of fading information**, the optimal strategy is to associate to the nearest BS of the best technology.*

Examples of Association Policies and Computations

Some examples of association policies.

- Optimal Association under no information.
- Optimal Association under nearest BS.
- Max-Ratio Association.
- Optimal Association under complete network information.

Max-Ratio Association

Optimal Association scheme great in theory.

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$$\pi_m = \left[\arg \max_{i \in [1,T]} \frac{r_1^i}{r_2^i}, 1\right]$$

Associate to the nearest BS of the technology yielding the maximum ratio.

(Oblivious to the statistical assumptions on the network.)

Max-Ratio Association

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This scheme trades off high signal power with that of interference power.

Max-Ratio Association - Asymptotic Optimality

Theorem - Let the noise powers be 0 and the reward function for all technologies are the same, i.e. $p_i(\cdot) = p(\cdot) \forall i$. Consider, the family of power law path-loss functions $\{l^{(\alpha)}(\cdot)\}_{\alpha > 2}$, where $l^{(\alpha)}(x) = x^{-\alpha}$. Let k be any integer larger than or equal to 2. If the information at the UE is the distance to the nearest k BS of each technology, i.e. $\mathcal{F}_I = \sigma(\cup_{i=1}^{T}(r_1^i, \cdots, r_k^i))$, then Max-Ratio is asymptotically almost surely optimal i.e.

$$\pi_\alpha^* \xrightarrow{\alpha \to \infty} \left[\arg \max_{i \in [1,T]} \frac{r_1^i}{r_2^i}, 1 \right] \text{ a.s.}$$

Max-Ratio Association - Finite Scale Behavior

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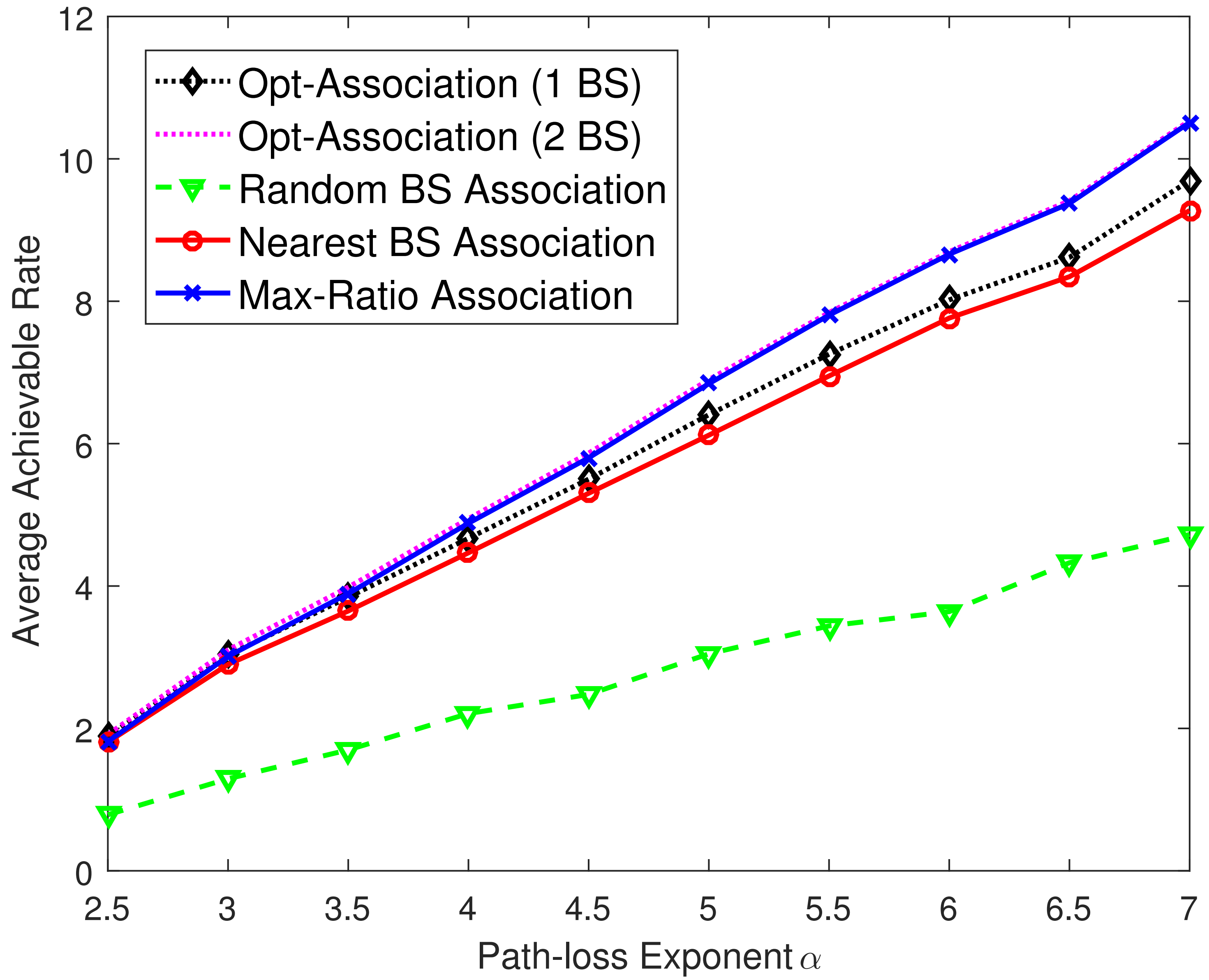

For $\alpha \geq 5$, Max-Ratio is nearly optimal.

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Denote by $f_i(\cdot)$ as the PDF of the random variable $\pi_i(\mathbf{r}_i, \lambda_i)$ and by $F_i(\cdot)$ as the associated CDF.

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Denote by $f_i(\cdot)$ as the PDF of the random variable $\pi_i(\mathbf{r}_i, \lambda_i)$ and by $F_i(\cdot)$ as the associated CDF.

Randomness

Formulas for Performance - Generalized Association

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Randomness

__Example__ - The Max-Ratio and the Optimal Association fit into this performance evaluation framework.

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$f_i(\cdot)$, $F_i(\cdot)$ The PDF and CDF of $\pi_i(\mathbf{r}_i, \lambda_i)$

Theorem -

$$\mathcal{R}_I^{\pi} = \sum_{i=1}^{T} \int_{\mathbf{r} \in \mathbb{R}^l} \mathbb{E}[p_i(SINR_0^{i, j_i(\mathbf{r}, \lambda_i)}) | \mathbf{r}] f_i(\mathbf{r}) \prod_{j=1, j \neq i}^{T} F_j(\pi_j(\mathbf{r}, \lambda_j)) d\mathbf{r}$$

Formulas for Performance - Generalized Association

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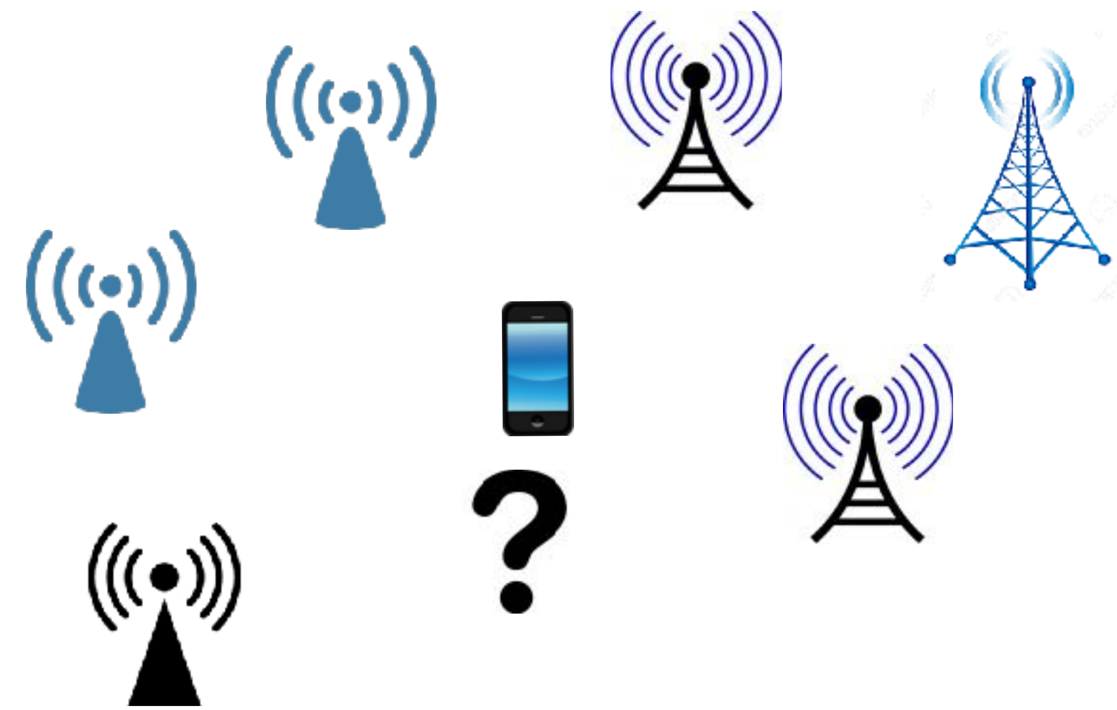
Corollary -

Closed form formulas for Max-Ratio and certain optimal association schemes.

Conclusions

Provide a systematic framework to ***design*** and evaluate association schemes that exploits the available information and technology diversity



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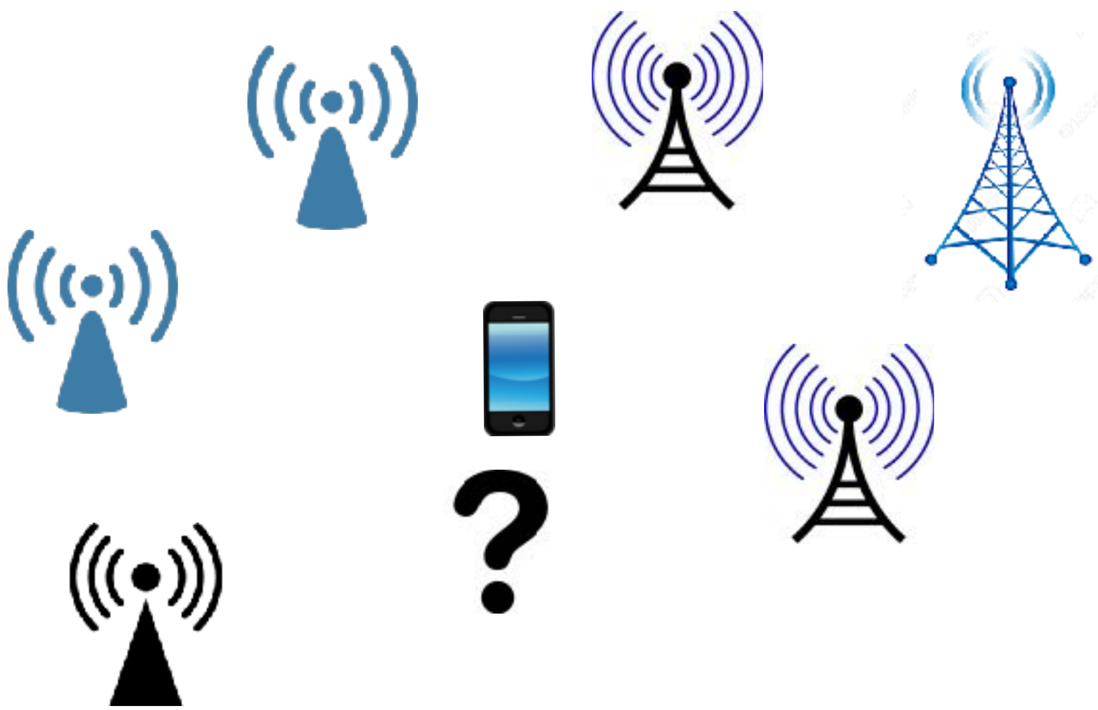

Max-Ratio Algorithm -

A simple and *almost* optimal algorithm for association.

Thank you for your time.